# Commentary on Euclid's Elements, Book I

Proclus

## DEFINITION I.

A point is that which has no PARTS.

That geometry, according to the transition which takes place from things more composite to such as are more simple, runs from body, which is diffused into distance by three dimensions, to a superficies by which it is bounded; but from superficies to a line, the boundary of superficies; and from a line to a point destitute of all dimension, has been often said, and is perfectly manifest. But because these terms, in many places, on account of their simplicity, appear to be more excellent than the nature of composites; but in many, as when they subsist in things which they terminate, they are similar to accidents, it is necessary to determine in what genera of beings each of these may be beheld [The present Comment, and indeed most of the following, eminently evinces the truth of Kepler's observation, in his excellent work, De Harmonia Mundi, p. 118.

For, speaking of our author's composition in the present work, which he every where admires and defends, he remarks as follows, "oratio fluit ipsi torrentis instar, ripas inundans, et cæca dubitationum vada gurgitesque occultans, dum mens plena majestatis tantarum rerum, luctatur in angustiis linguae, et conclusio nunquam sibi ipsi verborum copia satisfaciens, propositionum simplicitatem excedit." But Kepler was skilled in the Platonic philosophy, and appears to have been no less acquainted with the great depth of our author's mind than with the mangificence and sublimity of his language. Perhaps Kepler is the only instance among the moderns, of the philosophical and mathematical genius being united in the same person.] I say then, that such things as are destitute of matter, which subsist in separate reasons, and in those forms which are placed under themselves, are always alloted a subsistence of more simple essences, superior to the subsistence of such as are more composite.

On this account, both in intellect, and in the ornaments, as well of the middle kind as among those peculiar to the soul, and in natures themselves, the terms which proximately vivify bodies, excel according to essence the things which are terminated; and are more impartible, more uniform, and more primary than these. For in immaterial forms, unity is more perfect than multitude; that which is impartible, than that which is endued with unbounded progression;

and that which terminates, than that which receives bound from another. But such things as are indigent of matter, and abide in others, and degenerate from the perfection of their essence, which are scattered about subjects, and have an unnatural union, are allotted more composite reasons, prior to such as are more simple. Hence, things which appear in the phantasy invested with form, and the matter of the figures which the phantasy contains, and whatever in sensibles is generated by nature, have, in a preceding order, the reasons of the things terminated; but the reasons which terminate, in a following and adventitious rank [That is the reason of a triangular figure (for instance) in the phantasy, or triangle itself, is superior to the triangular nature participated in that figure.] For lest that which is distributed into three dimensions, should be extended into infinite magnitude, either according to intelligence or sense, it was every way terminated by superficies.

And lest a plane superficies should conceal itself in an infinite progression, a line approaching opposed its diffusion, and gave bound to its indefinite extension. And, in like manner, a point limited the progressions of a line; composite natures deriving their subsistence from such as are simple. For this also is again manifest, that in separate forms the reasons of terms subsist in themselves, but not in those which are terminated; and abiding such as they are in reality, possess a power of constituting secondary natures. But, in inseparable forms they give themselves up to things which are terminated, reside in them, become, as it were, their parts, and are replenished with baser natures. On which account, that which is impartible is there endued with a partible essence, and that which is void of latitude is diffused into breadth. And terms are no longer able to preserve their simplicity and purity.

For since they abide in another, they necessarily change their own nature into the matter of their containing subject. Matter, indeed, disturbs the perfection of these, and causes the reason of a plane to become a profound plane; but obscuring the one dimension of a line, causes it to be every way partible; and gives corporeity to the indivisibility of a point, and separates it together with the natures which it terminates. For all these reasons falling into matter, the one kind from cogitation into intelligible matter, but the other from nature into that which is sensible, are replenished with their containing subjects; and depart from their own simplicity, into foreign compositions and intervals. But here a doubt arises how all these, existing in intellect and soul in an impartible manner, and without any dimension, are distributed into matter, some indeed, principally, but others on account of its nature?

Shall we say that there is a certain order in immaterial forms, so that some are allotted the first, some the middle, and others the last place; and that of forms some are more uniform, but that others are more multiplied; and that some have their powers collected together, but others tending into interval; and that some, again, border upon bound, but that others are proximate to infinity? For though all participate of these two principles, yet some originate from bound, but others from infinity, of which they more largely participate. Hence, a point is entirely impartible, since it subsists according to bound, yet it occultly contains an infinite power, by which it produces every interval, and the progression of all intervals, unfolds its infinite power. But body, and the reason of body, participates more of an infinite nature; on which account it is among the number of things terminated by another, and divisible in infinitum, according to all dimensions.

But the mediums between these, according to the distance of the extremes, are either among such as have an affluence of infinity: on which account they both terminate and are terminated. For, indeed, so far as they consist from bound, they are able to terminate others; but so far as they participate of infinity, they are indigent of termination from others. Hence, since a point is also a bound, it preserves its proper power in participation: but since it likewise contains infinity occultly, and is compelled to be every where present with the natures which it terminates, it resides with them infinitely. And, because among immaterial forms there was a certain infinite power capable of producing things distant from each other by intervals, a point is present with its participants in capacity. For infinity in intelligibles is the primary cause and prolific power of the universe; but in material natures it is imperfect, and is alone all things in dormant capacity.

And in short, those forms which, on account of their simplicity and impartibility, hold a superior rank among principles, preserve, indeed, (in conformity to their nature,) their own property in their participations, but become worse than more composite reasons. For matter is able to participate these more clearly, and to be prepared for their reception, rather than that of the most simple causes of beings. On which account, the vestiges of separate principles descend into matter; but the participations of those in a second and third order, become more conspicuous. Hence, matter participates more of the cause of body, than of a plane; and of this more than the form of a line; and of this still more than that of a point, which contains all these, and is the boundary of them all. For the reason of a point presides over this whole series, unites and contains all partible natures, terminates their progressions, produces them all by its infinite power, and comprehends them in its indivisible bound.

On which account also, in the images of immaterial forms, some are the boundaries of others; but a point is the limit of them all. But that we must not think with the Stoics, that these boundaries of bodies alone subsist from cogitation; but that there are certain natures of this kind among beings, which previously contain the demiurgical reasons of things, we shall be enabled to remember, if we regard the whole world, the convolutions of its parts, the centres of those convolutions, and the axes which penetrate through the whole of these revolving circles. For the centres subsist in energy, since they contain the spheres, preserve them in their proper state, unite their intervals, and bind and establish to themselves the powers which they possess. But the axes themselves being in an immoveable position, evolve the spheres, give them a circular motion, and a revolution round their own abiding nature.

And the poles of the spheres, which both terminate the axes, and bind in themselves the other convolutions, do they not perspicuously evince, that points are endued with demiurgical and capacious powers, that they are perfective of every thing distant by intervals, and are the sources of union, and an unceasing motion? From whence, indeed, Plato also says, that they have an adamantine subsistence; shewing by this, the immutable, eternal, and stable power of their essence, ever preserving itself in the same uniform mode of existence. He adds too, that the whole spindle of the Fates, is turned about these, and leaps round their coercive union. But other more recondite and abstruse discourses affirm, that the demiurgus presides over the world, seated in the poles, and, by his divine love, converting the universe to himself. But the Pythagoreans thought that the pole should be called the Seal of Rhea; because the zoogonic, or vivific goddess, pours through these into the universe, and inexplicable and efficacious power.

And the centre they called the prison of Jupiter: because, since Jupiter has placed a demiurgical guard in the bosom of the world, he has firmly established it in the midst. For, indeed, the centre abiding, the universe possesses its immoveable ornament, and unceasing convolution: and the gods who preside over the poles, obtain a power collective of divisible natures, and unific of such as are multiplied: and those who are allotted the government of the axes, restrain and eternally evolve their perpetual convolutions. And, if it is lawful to offer our own opinion on this subject, the centres and poles of all the spheres are the symbols of the conciliating gods, shadowing forth their imperceptible and unifying composition. But the axes express the coherencies of the universal ornaments; and are endued with a power of comprehending the mundane integrities and periods, in the same manner as their presiding deities, of such as are intellectual.

But the spheres themselves are images of the gods, called perfectors of works, copulating the principle with the end, and excelling all figures in simplicity, similitude, and perfection. But we have been thus prolix, that we might evince the power of impartibles, and of the terms which the world contains, and that so far as they bear an image of primary and most principal causes, there are allotted the most excellent order in the universe. For centres and poles are not of the same kind with things which are terminated; but they subsist in energy, and possess an essence, and perfect power, which pervades through all partible natures. But many beholding those terms which imperfectly subsist in terminated essences, consider them as endued with a slender subsistence; and some indeed say, that they are alone separated from sensibles by thought; but others, that they have an essence no where but in our thoughts.

However, since the forms of all these are found both in the nature of intellect, in the ornaments of soul, in the nature of things, and in inferior bodies, let us consider how, according to the order they contain, they subsist in the genera of beings. And indeed, all of them pre-exist in intellect, but in an impartible and uniform manner: so that they all subsist according to one form, the reason of a point, which exists occultly and impartibly. But they all subsist in soul according to the form of a line: on which account Timæus also composes the soul from right and circular lines: for every circle is a line alone. But they all subsist in natures, according to the reason of a plane; and on this account, Plato commands us to manifest those natural reasons, which are endued with a power of constituting bodies by a plane. And the resolution of bodies into planes leads us to the proximate cause of appearances.

Lastly, they all subsist in bodies, but in a corporeal manner; since all forms have their being in these, according to the partible nature of bodies. Hence, all of them appear every where, and each according to its proper order; and diversity arises from predenominating power. The point, indeed, is every where impartible, and when that which is divisible into parts, excels according to the diminution of beings, it vindicates to itself, an illustrious subsistence of partible natures. And sometimes the point is entirely superior, according to the excellence of cause; but sometimes it is connected with divisibles, and sometimes it is allotted in them and adventitious existence; and, as if swallowed up by the partition of the lowest natures, loses its own proper impartibility. As, therefore, with respect to the monad, one is the mother of number, but the other is as matter spread under, and the receptacle of numbers; and each of them is a principle, (yet neither of them is number), but in a different respect: in the same manner a point also, is partly the parent and author of magnitudes; but is partly a principle in another respect, and not according to a generative cause.

But is a point, then, the only impartible? Or may we affirm this of the now in time, and of unity in numbers? Shall we not say, that to the philosopher, indeed, discoursing concerning the universality of things, it is proper to behold every thing, however falling under distribution; but that to him who is endued with the science of particulars, who produces his contemplation from certain definite principles, and runs back even to these, but very little scrutinizes the progressions of beings, it is requisite to attempt, consider, and treat concerning that impartible nature alone, which regards his first principles; and to behold that simplicity which presides over all the particular subjects of his knowledge? In consequence of this reasoning, therefore, a point alone, according to the geometric matter, is destitute of partition; but unity according to that which is arithmetical.

And the reason of a point, however in some other respects it may be imperfect, yet is perfect in the present science. For, indeed, the physician also says, that the elements of bodies are fire and water, and things similar to these; and as far as to these the resolution of bodies proceeds. But the natural philosopher passes on to more simple elements; and the one defines an element simple as to sense, but the other simple as to reason; and both of them properly as to their peculiar science. We must not, therefore, think that the definition of a point is faulty, nor determine it as imperfect; for so far as pertains to the geometric matter, and its principles, it is sufficiently delivered. This alone, indeed, is wanting to its completion, that the definition does not clearly say, that which is impartible with me is a point; and my principle, and that which I contain as most simple, is nothing else than this.

And after this manner it is proper to hear the geometrician addressing us. Euclid, therefore, from a negation of parts, declares to us a principle, leading to the theory of its whole subject nature. For negative discourses are proper to principles, as Parmenides teaches us, who delivers the doctrine concerning the first and last cause, by negations alone. Since every principle consists of an essence different from its flowing consequents; and the negations of these exhibit to us the property of their source. For that it is, indeed, the cause of these, yet at the same time has nothing in common with these, becomes perspicuous from a doctrine of this kind. But here a doubt may arise, how, since the phantasy receives all things invested with forms, and in a partible manner, the geometrician beholds in it the point destitute of parts? For it is not because they are reasons existing in cogitation, but the phantasy receives the resemblances of intellectual and divine forms according to its own proper nature, exhibiting in its shadowy bosom the forms of formless natures, and clothing with figure things entirely free from the affectations of figure.

To this ambiguity we must say, that the species of imaginative motion is neither alone partible, nor impartible; but that it proceeds from the impartible to the partible, and from the formless nature to that which is expressed by form. For it was partible alone, it could not preserve in itself many impressions of forms, since the subsequent would obscure the pre-existent figures: for no body can contain at once, and according to the same situation, a multitude of figures; but the former will be blotted out by the succession of the latter. But if it was alone impartible, it would not be inferior to cogitation, and to soul, which surveys all things in an impartible manner. Hence, it is necessary that it should indeed begin from an impartible according to its motion, and from thence draw forth the folded and scattered form of every thing falling under cogitation, and penetrating to its shadowy receptacle: but, that it should at length end in form, figure and interval.

And if it be allotted a nature of this kind, it will, after a certain manner, contain an impartible essence: and a point, according to this, must be said to have its principal subsistence: for the form of a line is contracted in the phantasy according to this. Hence, because it possesses a twofold power, impartible and partible, it will indeed contain a point in an impartible, and intervals in a partible manner. But as the Pythagoreans define a point to be unity having position, let us consider what they mean. That numbers, indeed, are more immaterial and more pure than magnitudes, and that the principle of numbers is more simple than the principle of magnitudes, is manifest to every one: but when they say that a point is unity endued with position, they appear to me to evince that unity and number subsist in opinion: I mean monadic number.

On which account, every number, as the pentad and the heptad, is one in every soul, and not many; and they are destitute of figure and adventitious form. But a point openly presents itself in the phantasy, subsists, as it were, in place, and is material according to intelligible matter. Unity, therefore, has no position, so far as it is immaterial, and free from all interval and place: but a point has position, so far as it appears seated in the bosom of the phantasy, and has a material subsistence. But unity is still more simple than a point, on account of the community of principles. Since a point exceeds unity according to position; but appositions in incorporeals produce diminutions of those natures, by which the appositions are received.

## DEFINITION II.

A Line is a Length without Breadth.

A line obtains the second place in the Definitions, as it is by far the first and most simple interval, which the geometrician calls a length, adding also without breadth; since a line, in respect of a superficies, ranks as a principle. For he defines a point, as it is the principle of all magnitudes, by negation alone; but a line, as well by affirmation as by negation. Hence it is a length, and by this exceeds the impartibility of a point; but it is without breadth, because it is separated from other dimensions. For, indeed, every thing which is void of breadth, is also destitute of bulk, but the contrary is not true, that every thing void of bulk is also destitute of breadth. Since, therefore, he has removed breadth from a line, he has also removed at the same time bulk. On which account he does not add, that a line also has no thickness, because this property is consequent to the notion of being without breadth.

But it is defined by others in various ways: for some call it the flux of a point, but others a magnitude contained by one interval. And this definition, indeed, is perfect, and sufficiently explains the essence of a line; but that which calls it the flux of a point, appears to manifest its nature from its producing cause; and does not express every line, but alone that which is immaterial. For this is produced by a point, which though impartible itself, is the cause of being to partible natures. But the flux of a point, shews its progression and prolific power, approaching to every interval, receiving no detriment, perpetually abiding the same, and affording essence to all partible magnitudes. However, these observations are known, and manifest to every one. But we shall recall into our memory, discourses more Pythagorical, which determine a point as analogous to unity, a line to the duad, a superficies to the triad, and body to the tetrad. [Yet when we compare those which receive interval together, we shall find a line monadic, but a superficies dyadic, and a solid body triadic.] From whence also, Aristotle says, that body is perfected by the ternary number.

And, indeed, this is not wonderful, that a point, on account of its impartibility, should be assimilated to unity; but that things subsequent to a point, should subsist according to numbers proceeding from unity, and should preserve the same proportion to a point, as numbers to unity; and that every one should participate of its proximate superior, and have the same proportion to its kindred, and following degree, as the superior to this, which is the immediate consequent. [For example, that a line has the order of the duad with respect to the point, but of unity to a superficies; and that this last has the relation of a triad to the point, but of the duad to a solid.] And on this account, body is tetradic, with respect to a point, but triadic as to a line. Each order, therefore, has its proportion; but the order of the Pythagoreans is the more principal, which receives its commencement from an exalted source, and follows the nature of beings.

For a point is indeed twofold; since it either subsists by itself, or in a line; in which last respect also, since as a boundary it is alone and one, neither having a whole nor parts, it imitates the supreme nature of beings. On which account too, it was placed in a correspondent proportion to unity. For as the oracle says, Unity is there first, where the paternal unity abides. But a line is the first endued with parts and a whole, and it is monadic because it is distant by one interval only; and dyadic on account of its progression: for if it be infinite, it participates of the indefinite duad; but if finite, it requires two terms, from whence and to what place; since on account of these it imitates totality, and is allotted an order among totals. For unity, according to the oracle, is extended, and generates two; and this produces a progression into longitude, together with that which is distant extendedly, and with one interval, and the matter of the duad.

But superficies, since it is both a triad and duad, as also the receptacle of the primary figures, and that which receives the first form and species, is in a certain respect similar to the triadic nature, which first terminates beings; and to the duad, by which they are divided and dispersed. But a solid, since it has a triple distance, and is distinguished by the tetrad, which is endued with a power of comprehending all reasons, is reduced to that order in which the distinction of corporeal ornaments appears; as also the division of the universe into three parts, together with the tetradic property, which is generative and female. And these observations, indeed, might be more largely discussed, but for the present, must be omitted. Again, the discourse of the Pythagoreans, not undeservedly, calls a line, which is the second in order, and is constituted according to the first motion from an impartible nature, dyadic.

And that a point is posterior to unity, a line to the duad, and a superficies to the triad, Parmenides himself shews, by first of all taking away multitude form one by negation, and afterwards the whole. Because, if multitude is before that which is a whole, number also will be prior to that which is continuous, and the duad to the line, and unity to the point: since the epithet not many, belongs to unity which generates multitude, but to the point, the term not a whole, is proper, because it produces a whole; for this is said to have no part. And these things are affirmed of a line, while we more accurately contemplate its nature. But we should also admit the followers of Apollonius, who say, that we obtain the notion of a line, when we are ordered to measure the lengths alone, either of ways or walls; for we do not then subjoin either breadth or bulk, but only make one distance the object of our consideration.

In the same manner we perceive superficies, when we measure fields; and a solid, when we take the dimensions of wells. For then, collecting all the distances together, we say, that the space of the well is so much, according to length, breadth, and depth. But a line may become the object of our sensation, if we behold the divisions of lucid places from those which are dark, and survey the moon when dichotomized: for this medium has no distance which respect to latitude; but is endued with longitude, which is extended together with the light and shadow.

## DEFINITION III.

But the Extremities of a Line are Points.

Every composite receives its bound from that which is simple, and every thing partible from that which is impartible; and the images of these openly present themselves in mathematical principles. For when it is said that a line is terminated by points, it seems manifestly to make it of itself infinite, because, on account of its proper progression, it has no extremity. As, therefore, the duad is terminated by unity, and reduces its own intolerable boldness under bound, when it is restrained in its comprehensive embrace: so a line also is limited by the points which it contains. For, since it is similar to the duad, it participates of a point having the relation of unity, according to the nature of the duad. Indeed, in imaginative, as well as in sensible forms, the points themselves terminate the lines in which they reside. But in immaterial forms, the reason of the impartible point pre-exists separate and apart; but when proceeding from thence by far the first of all, by determining itself with interval, moving itself, and flowing in infinite progression, and imitating the indefinite duad, it is restrained indeed, by its proper principle, is united by its power, and on every side seized by its coercive bound.

Hence it is, at the same time, both infinite and finite: infinite, indeed, according to its progression; but finite according to its participation of a terminating cause. So that, when it approaches to this cause, it is detained in its comprehension, and is terminated according to its union. Hence too, in the images of incorporeal forms, a point is said to terminate a line, by occupying its beginning and end. Bound, therefore, in immaterials, is separated from that which is bounded: but here it is twofold; for it subsists in that which is terminated. And this affords a wonderful symptom, that forms, indeed, abiding in themselves, precede their participants according to cause; but when giving themselves up to their subordinate natures, subsist according to their diversified properties: since they are multiplied and distributed together with these, and receive the division of their subjects.

Besides, this also must be previously received concerning a line, that our geometrician uses it in a threefold acceptation. As terminated on both sides, and finite; as in the problem which says, Upon a given terminated right line to construct an equilateral triangle. And as partly infinite and partly finite; as in the problem which commands us from three right lines, which are equal to three given right lines, to construct a triangle; for in the construction of the problem, he says, Let there be placed a certain right line, on the one part finite, but on the other part infinite. And again, a line is received by Euclid as on both sides infinite; as in the problem which says, Upon a given infinite right line, from a given point, which is not in that line, to let fall a perpendicular. But, besides this, the following doubts, since they are worthy of solution, must not be omitted.

How are points called extremities of a line? and of what line, since they can neither be the bounds of one that is infinite, nor of every finite? For there is a certain line, which is both finite, and has not points for its extremities. And such is a circular line, which returns into itself, and is not bounded by points, like a right line. And such also is the ellipsis, or line like a shield. Is it therefore requisite to behold a line, considered as a line? for we must receive a certain circumference, which is terminated by points, and a part of the elliptic line; having, in like manner, its extremities bounded by points. But every circular and elliptic line, assumes to itself another certain property, by which it is not line alone, but is also endued with a power of perfecting figure. Lines, themselves, therefore, have their extremities terminated by points; but those which are effective of such like figures, return into themselves. And, indeed, if you conceive them to be described, you will also find how they are bounded by points; but if you receive them already described, and connect the end with the beginning, you can no longer behold their extremes.

## DEFINITION IV.

A Right Line, is that which is equally situated between its bounding Points.

Plato, establishing two most simple and principal species of lines, the right and the circular, composes all the rest from the mixture of these; I mean such as are called curve lines, some of which are formed from planes, but others subsist about solids; and whatever species of curve lines are produced by the sections of solids. And it seems, indeed, that a point (if it be lawful so to speak) bears an image of the one itself, according to Plato: for unity has no part, as he also shews in the Parmenides. But, because after unity itself there are three hypostases, or substances, bound, infinite, and that which is mixed from these, the species of lines, angles, and figures, which subsist in the nature of things originate from thence. And, indeed, a circumference and a circular angle, and a circle among plane figures, and a sphere among solids, are analogous to bound.

But a right line corresponds to infinity, according to all these; for it properly belongs to all, if it is beheld as existing in each. But that which is mixed in all these, is analogous to the mixt which subsists among intelligibles. For lines are mixed, as those which are called spirals. And angles, as the semi-circular and cornicular. And plane figures, as segments and apsides; but solids, as cones and cylinders, and others of that kind. Bound, therefore, infinite, and that which is mixed, are participated by all these. But Aristotle likewise assents to Plato; for every species of lines, says he, is either right or circular, or mixed from these two. From whence also there are three motions, one according to a right line; the other circular; and the third mixed. But some oppose this division, and say that there are not two simple lines alone, but that there is a certain third line given, i. e. a helix or spiral, which is described about a cylinder, when, whilst a right line is moved round the superficies of the cylinder, a point in the line is carried along with an equal celerity.

For by this means, a helix, or circumvolute line, is produced, which adapts all the parts of itself to all, according to a similitude of parts, as Apollonius shews in his book concerning the Cochlea; which passion, among all spirals, agrees to this alone. For the parts of a plane helix are dissimilar among themselves; as also of those which are described about a cone and sphere. But the cylindric spiral alone, consists of similar parts in the same manner as a right and circular line. Are there, then, three simple lines, and not two only? To which doubt we reply, that a helix of this kind is, indeed, of similar parts, as Apollonius teaches, but is by no means simple; since among natural productions, gold and silver are composed of similar parts, but are not simple bodies. But the generation of the cylindric helix evinces that its mixture is from things simple; for it originates while a right line is circularly moved round the axis of the cylinder, a point at the same time flowing along in the right line.

Two simple motions, therefore, compose its nature; and, on this account, it is among the number of mixt lines, and not among such as are simple: for that which is composed from dissimilars is not simple, but mixt. Hence, Geminus, with great propriety, when he admits that some simple lines may be produced from many motions, does not grant that every such line is mixt; but that alone, which arises from dissimilar motions. For if you conceive a square, and two motions which are performed with an equal celerity, one according to the length, but the other according to the breadth, a right line or the diameter will be produced; but the right line will not, on this account, be mixed: for no other line precedes it, formed by a simple motion, as we asserted of the cylindric helix. Nor yet, if you suppose a right line, moving in a right angle, and by a bisection to describe a circle, is the circular line, on this account, produced with mixture: for the extremities of that which is moved after this manner, since they are equally moved, will describe a right line; and the bisection, since it is unequally devolved, will delineate a circle; but the other points will describe an ellipsis: On which account, the generation of a circular line is the consequence of that inequality of lation arising from the bisection; because a right line was supposed to be moved in a right angle, but not in a natural manner.

And thus much concerning the generation of lines. But it seems, that of the two simple lines, the right and the circular, the right line is the more simple; for in this, dissimilitude cannot be conceived, even in opinion. But in the circular line, the concave and the convex, indicate dissimilitude. And a right line, indeed, does not infer a circumference according to thought; but a circumference brings with it a right line, though not according to its generation, yet with respect to its centre. But what if it should be said that a circumference requires a right line to its construction! For if either extreme of a right line remains fixt, but the other is moved, it will doubtless describe a circle, whose centre will be the abiding extreme of the right line. Shall we say that the generator of the circle is the point which is carried about the abiding point, but not the right line itself?

For the line only determines the distance, but the point composes the circular line, while it is moved in a circular manner: but of this enough. Again, a circumference appears to be proximate to bound, and to have the same proportion to other lines, as bound to the universaility of things. For it is finite, and is alone among simple lines perfective of figure. But a right line is proximate to infinity; for its capacity of infinite extension never fails: and as all the rest are produced from bound and infinite, in the same manner from the circular and right line, every mixt genus of lines is composed, as well of planes as of those which consist in solid bodies. And on this account, the soul also previous assumed into herself the right and circular according to her essence, that she might moderate all the co-ordination of infinite, and all the nature of bound, which the world contains.

By a right line, indeed, constituting the progression of these principles into the universe; but by a circular line, their return to their original source: and by the one, producing all things into multitude; but by the other, collecting them into one. And not only the soul, but he also who produced the soul, and endued her with these powers, contains in himself both these primary causes. For when he previously assumed the beginning, middle, and end of all things, he terminated right lines (says Plato), by a circular progression according to nature. And proceeding to all things by provident energies, and returning to himself, he established himself, says Timæus, after his own peculiar manner. But a right line is the mark or symbol of a providence, indeclinable, incapable of perversion, immaculate, never-failing, omnipotent, and present to all beings, and to every part of the universe.

But a circumference, and that which environs, is the symbol of an energy retiring into union with itself, and which rules over all things according to one intellectual bound. When, therefore, the demiurgus of the universe had established in himself these two principles, the right and the circular line, and had given them dominion, he produced from himself two unities; the one, indeed, energizing according to the circular line, and being effective of intellectual essences; but the other according to the right line, and affording an origin to sensible natures. But because the soul is allotted a middle situation between intellectuals and sensibles, so far, indeed, as she adheres to an intellectual nature, she energizes according to the circle; but so far as she presides over sensibles, she provides for their welfare according to the right line: and thus much concerning the similitude of these forms to the universality of things.

But Euclid, indeed, has properly delivered the present definition of a line; by which he shews that a right line alone occupies a space equal to that which is situated between its points: for as much as is the distance of one point from another, so great is the magnitude of the lines terminated by the points. And this is the meaning of being equally situated between its extremes. For if you take two points in a circumference, or in any other certain line, the space of line which is included between these, exceeds their distance from each other; and every line, besides a right one, appears to suffer this property. Hence, according to a common conception, the vulgar also say, that he who walks by a right line, performs only a necessary journey: but that they necessarily wander much, who do not proceed in a right line. But Plato thus defines it; a right line is that whose middle parts darken its extremes.

For this passion necessarily attends things which have a direct position; but it is not necessary that things situated in the circumference of a circle, or in another interval, should be endued with this property. Hence, the astrologers also say, that the sun then suffers an eclipse when that luminary, the moon, and our eye are in one right line; for it is then darkened through the middle position of the moon between us and its orb. And perhaps, the passion of a right line will evince, that in things which are, according to processions emanating from causes, the mediums are endued with a power of dividing the distance of the extremes, and their mutual communication with each other. As also, according to regressions, such things as are distant from the extremes, are converted by mediums to their primary causes. But Archimedes defines a right line the least of things having the same bounds.

For since, according to Euclid, a right line is equally situated between its points, it is on this account, the least of things having the same bounds: for if a less line could be given, it would not lie equally between its extremes: but all the other definitions of a right line, fall into the same conclusions; as for instance, that it is constituted in its extremities, and that one part of it is not in its subject plane, but another, in one more sublime: and that all its parts similarly agree to all: and that its extremes abiding, it also abides. Lastly, that it does not perfect figure, with one line similar in species to itself: for all these definitions express the property of a right line, which it possesses from the simplicity of its essence, and from its having one progression the shortest of all from one extremity to another. And thus much concerning the definitions of a right line.

But again, Geminus divides a line first into an incomplete and composite; calling a composite, that which is refracted, and forms an angle; but all the rest of them, he denominates incomposites. Afterwards, he divides a composite line into that which produces figure, and that which may be infinitely extended. And he calls that which produces figure, a circular line, and the line of a shield, and that which is similar to an ivy leaf; but that which is not effective of figure, the section of a rectangular and obtuse angular cone, the line similar to a shell, the right line, and all of that kind. And again, after another manner, of the incomposite line, one sort is simple, but the other mixt. And of the simple, one produces figure, as the circular; but the other is indefinite, as the right line. But of the mixt, one subsists in planes, but the other in solids.

And of that which is in planes, one coincides in itself, as the figure of the ivy leaf, which is called the cissoid; but the other may be produced in infinitum, as the helix. but of that which is in solids, one may be considered in the sections of solids; but the other as consisting about the solids themselves. For the helix, indeed, which is described about a sphere or a cone, consists about solids; but conic, or spiral sections are generated from a particular section of solids. But, with respect to these sections, the conic were invented by Mænechmus, which also Eratosthenes relating, says, "Nor in a cone Mænechmian ternaries divide."

But the spiric by Perseus, who composed an epigram on their invention, to this purpose, "When Perseus had invented three sparal lines in five sections, he sacrificed to the gods on the occasion." And the three sections of a cone, are the parabola, hyperbola, and ellipsis: but of spiral sections, one kind is twisted and involved, like the fetlock of a horse; but another is dilated in the middle, and deficient in each extremity: and another which is oblong, has less space in the middle, but is dilated on each side. But the multitude of the other mixt lines is infinite. For there is an innumerable multitude of solid figures, from which there are constituted multiform sections. For a right line, while it is circularly moved, does not make a certain determinate superficies, nor yet conical, nor conchoidal lines, nor circumferences themselves. Hence, if these solids are multifariously cut, they will exhibit various species of lines. Lastly, of those lines which consist about solids, some are of similar parts, as the helixes about a cylinder; but others of dissimilar parts, as all the rest. From these divisions, therefore, we may collect, that there are only three lines of similar parts, the right, the circular, and the cylindric helix. The two simple ones, indeed, existing in a plane, but the one mixt, about a solid. And this Geminus evidently demonstrates, when he shews, that if two right lines are extended from one point, to a line of similar parts, so as to make equal angles upon that line, they shall be equal to each other. And the demonstrations of this may be received by the studious, from his volumes; since in these he delivers the origin of spiral, conchoidal, and cissoidal lines. But we have barely related the names and divisions of these lines, for the purpose of exciting the ingenious to their investigation; as we think, that an accurate enquiry after the method of detecting the properties of each, would be superfluous in the present undertaking: since the geometrician only unfolds to us in this work, simple and primary lines, i. e. the right line, in the present definition; but the circular line, in the tradition of a circle. For he then says, that the line terminating the circle, is the circumference. But he makes no mention of mixt lines, though he was well acquainted with mixt angles, I mean, the semicircular and cornicular: as also with plane mixt figures, i. e. segments and sectors; and with solids, viz. cones and cylinders.

Of each of the rest, therefore, he delivers three species; but of lines only two, i. e. the right and circular: for he thought it requisite in discourses concerning things simple, to assume simple species; and all the rest are more composite than lines. Hence, in imitation of the geometrician, we also shall terminate their explanation with simple lines.

## DEFINITION V.

A superficies is that which has only Length and Breadth.

After a point and a line, a superficies is placed, which is distant by a twofold interval, length and breadth. But this also remaining destitute of thickness or bulk, possesses a nature more simple than body, which is distant by a triple dimension. On which account the geometrician adds to the two intervals the particle only, because the third interval does not exist in superficies. And this is equivalent to a negation of bulk, as here also he shews the excellency of superficies compared to a solid with respect to simplicity, by negation, or by an addition equivalent to negation: but the diminution which it possesses, if compared with the preceding terms, by the affirmations themselves. But others define a superficies to be the boundary of body, which is almost affirming the same as the definition of Euclid; since that which terminates is exceeded in one dimension, by that which is terminated. And others, a magnitude different by two intervals. Lastly, others declaring the same affection, form its assignation in a somewhat different manner. But they say we have a knowledge of superficies when we measure fields, and distinguish their extremities according to length and breadth; but that we receive a certain sensation of it, when we behold shadows. For as they are without bulk, because they cannot penetrate into the interior part of the earth, they have only length and breadth. But the Pythagoreans say, that it is assimilated to the triad; because the ternary is by far the first cause to all the figures; which a superficies contains. For a circle, which is the principle of orbicular figures, occultly possesses the ternary, by its centre, interval, and circumference. But a triangle, which ranks as the first among all right-lines figures, on every side evinces that it is enclosed by the triad, and receives its form from its perfect nature.

## DEFINITION VI.

The Extremities of a SUPERFICIES are Lines.

From these also, as images, we may understand, that things more simple procure bound and an end to every one of their proximate natures: for soul perfects and determines the operations of nature; and nature the motion of bodies. And prior to these, intellect measures the convolutions of soul; and unity the life of intellect; for that is the measure of all. Just as in these also, a solid is terminated by a superficies; but a superficies by a line; and a line by a point; for that is the boundary of them all. Hence the line existing uniformly in immaterial forms and impartible reasons, terminates and restrains the various motion of a superficies in its progression, and proximately unites its infinity. But in the images of these, when that which bounds supervenes that which is bounded, it causes, by this means, its limitation and bound. But if it should be enquired how lines are the extremities of every superficies, since they are not the extremes of every finite figure; for the superficies of a sphere is terminated indeed, yet not by lines, but by itself? In answer to this, we must say, that by receiving a superficies so far as it is distant by a two-fold interval, we shall find it terminated by lines according to length and breadth. But if we behold a spherical superficies, we must receive it as that which is endued with figure; which possesses another quality, and conjoins the end with the beginning; and loses its two extremities in the comprehensive embraces of one: and this one extremity subsists in capacity only, and not in energy.

## DEFINITION VII.

A Plane SUPERFICIES is that which is equally situated between its bounding Lines.

It was not agreeable to the ancient philosophers to establish a plane species of superficies; but they considered superficies in general, as the representative of magnitude, which is distant by a two-fold interval. For thus the divine Plato says, that geometry is contemplative of planes, opposing it in division to stereometry, as if a plane and a superficies were the same. And this was likewise the opinion of the demoniacal Aristotle. But Euclid and his followers consider superficies as a genus, but a plane as its species, in the same manner as rectitude of a line. And on this account he defines a plane separate from a superficies, after the similitude of a right line. For he defines this last as equal to the space, placed between its points. And in like manner, he says, that two right lines being given, a plane superficies occupies a place equal to the space situated between those two lines.

For this is equally situated between its lines; and others also explaining the same boundary, assert that it is constituted in its extremities. But others define it as that to all the parts of which a right line may be adapted. But perhaps others will say, that it is the shortest of superficies, having the same boundaries; and that its middle parts darken the extremities; and that all the definitions of a right line may be transferred into a plane superficies, by only changing the genus: since a right, circular, and mixt line, commencing from lines, arrive even at solids, as we have asserted above; for they are proportionally, both in superficies and solids. Hence also, Parmenides says, that every figure is either right, circular, or mixt. But if you wish to consider the right in superficies, take a plane, to which a right line agrees in various ways; but if a circular receive a spherical superficies; and if a mixt, a conic or cylindric, or some one of that genus.

But it is requisite (says Geminus) since a line, and also a superficies is called mixt, to know the measure of mixture, because it is various. For mixture in lines, is neither by composition, nor by temperament only: since, indeed, a helix is mixed, yet one part of it is not straight, and another part circular, like those things which are mixed by composition: nor if a helix is cut after any manner, does it exhibit an image of things simple, such as those which are mixed through temperament; but in these the extremes are, at the same time, corrupted and confused. Hence, Theodorus the mathematician, does not rightly perceive, in thinking that this mixture is in lines. But mixture in superficies, is neither by composition, nor by confusion; but subsists rather by a certain temperament. For conceiving a circle in a subject plane, and a point on high, and producing a right line from the point to the circumference of the circle, the revolution of this line will produce a conical superficies which is mixt.

And we again resolve it into its simple elements, by a parallel section: for by drawing a section between the vertex and the base, which shall cut the plane of the generative right line, we effect a circular line. But the idea of lines, shews that the mode of mixture is not by temperament; for neither does it send us back to the simple nature of elements: on the contrary, when superficies are cut, they immediately exhibit to us their producing lines. The mode of mixture, therefore, is not the same in lines and superficies. But as among lines there were some simple, that is, the right and circular, of which the vulgar also possess an anticipated knowledge without any previous instruction; but the species of mixt lines require a more artificial apprehension: so among superficies, we possess an innate notion of those which are especially elementary, the plane and spherical; but science and its reason investigates the variety of those which are composed through mixture.

But this is an admirable property of superficies, that their mixture in generation is oftentimes produced from a circular line; and this also happens to a spiral superficies. For this is understood by the revolution of a circle remaining erect, and turning itself about the same point which is not its centre. And on this account, a spiral also is threefold; for its centre is either in a circumference, or within, or external to a circumference. If the centre is in the circumference, a continued spiral is produced: if within the circumference, an intangled one; if without, a divided one. And there are three spiral sections corresponding to these three differences. But every spiral line is mixt, although the motion from which it is produced is one and circular. And mixt superficies are produced as well from simple lines, (as we have said,) while they are moved with a motion of this kind, as from mixt lines.

Since, therefore, there are three conic lines, they produce four mixt superficies, which they call conoids. For a rectangular conoid, is produced from the revolution of the parabola about its axis: but that which is formed by the ellipsis, is called a spheroid; and if the revolution is made about the greater axis, it is an oblong; but if about the lesser a broad spheroid. Lastly, and obtuse-angled conoid is generated from the revolution of the hyperbola. But it is requisite to know, that sometimes we arrive at the knowledge of superficies from lines, and sometimes the contrary; for from conical and spiral superficies, we apprehend conical and spiral lines. Besides, this also must be previously received concerning the difference of lines and superficies, that there are three lines of similar parts (as we have already observed), but only two superficies, the plane and the spherical.

For this is not true of the cylindric, since all parts of the cylindric superficies cannot agree to all. And thus much concerning the differences of superficies, one of which the geometrician having chosen (I mean the plane), this also he has defined; and in this, as a subject, he contemplates figures, and their attendant passions: for this discourse is more copious in this than in other superficies: since, indeed, we may understand right lines, and circles, and helixes in a plane; also the sections of circles and right lines, contacts, and applications, and the constructions of angles of every kind. But in other superficies, all these cannot be beheld. For how in one that is spherical, can we apprehend a right line, or a right-lined angle? How, lastly, in a conic of cylindrical superficies, can we behold sections of circles or right lines?

Not undeservedly, therefore, does he both define this superficies, and discuss his geometrical concerns, by exhibiting every thing in this as in a subject; for from hence he calls the present treatise plane. And, after this manner, it is requisite to understand that which is plane, as projected and constituted before the eyes: but cogitation as describing all things in this, the phantasy corresponding to a plane mirror, and the reasons resident in cogitation as dropping their images into its shadowy receptacle.

## DEFINITION VIII.

A plane Angle, is the inclination of two Lines to each other in a Plane, which meet together, but are not in the same direction.

Some of the ancient philosophers, placing an angle in the predicament of relation, have said, that it is the mutual inclination of lines or planes to each other. But others, including this in quality, as well as rectitude and obliquity, say, that it is a certain passion of a superficies or a solid. And others, referring it to quantity, confess that it is a superficies or a solid. For the angle which subsists in superficies is divided by a line; but that which is in solids, by a superficies. But (say they) that which is divided by these, is no other than magnitude, and this is not linear, since a line is divided by a point; and therefore it follows that it must be either a superficies or a solid. But if it is magnitude, and all finite magnitudes of the same kind have a mutual proportion; all angles of the same kind, i. e. which subsist in superficies, will have a mutual proportion.

And hence, the cornicular will eb proportionable to a right-lined angle. But things which have a mutual proportion, may, by multiplication, exceed each other; and therefore it may be possible for the cornicular to exceed a right-lined angle, which it is well-known, is impossible, since it is shewn to be less than every right-lined angle. But if it is quality alone, like heat and cold, how is it divisible into equal parts? For equality, inequality, and divisibility, are not less resident in angles than in magnitudes; but they are, in like manner, essential. But if the things in which these are essentially inherent, are quantities, and not qualities, it is manifest that angles also are not qualities. Since the more and the less are the proper passions of quality, but not equal and unequal. On this hypothesis, therefore, angles ought not to be called unequal, and this greater, but the other less; but they ought to be denominated dissimilars, and one more an angle, but the other less.

But that these appelations are foreign from the essence of mathematical concerns, is obvious to every one: for every angle receives the same definition, nor is this more an angle, but that less. Thirdly, if an angle is inclination, and belongs to the category of relation, it must follow, that from the existence of one inclination, there will also be one angle, and not more than one. For if it is nothing else than the relation of lines or planes, how is it possible there can be one relation of lines or planes, but many angles? If, therefore, we conceive a cone cut by a triangle from the vertex to the base, we shall behold one inclination of the triangular lines in the semicone to the vertex; but two distinct angles: one of which is plane, I mean that of the triangle; but the other subsists in the mixt superficies of the cone, and both are comprehended by the two triangular lines.

The relation, therefore, of these, do not make the angle. Again, it is necessary to call an angle either quality or quantity, or relation; for figures, indeed, are qualities, but their mutual proportions belong to relation. It is necessary, therefore, that an angle should be reduced under one of these three genera. Such doubts, then, arising concerning an angle, and Euclid calling it inclination, but Apollonius the collection of a superficies, or a solid in one point, under a refracted line or superficies (for he seems to define every angle universally), we shall affirm, agreeable to the sentiments of our preceptor Syrianus, that an angle is of itself none of the aforesaid; but is constituted from the concurrence of them all. And that, on this account, a doubt arises among those who regard one category alone. But this is not peculiar to an angle, but is likewise the property of a triangle.

For this, too, participates of quantity, and is called equal or unequal; because it has to quantity the proportion of matter. But quality also, is present with this, in consequence of its figure (since triangles are called as well similar as equal); but it possesses this from one category, and that from another. Hence, an angle is perfectly indigent of quantity, the subject of magnitude. But it is also indigent of quality, by which it possesses, as it were, its proper form and figure. Lastly, it is indigent of the relation of lines terminating, or of superficies comprehending its form. So that an angle consists from all these, yet is not any one of them in particular. And it is indeed divisible, and capable of receiving equality and inequality, according to the quantity which it contains. But it is not compelled to admit the proportion of magnitudes of the same kind, since it has also a peculiar quantity, by which angles are also incapable of a comparison with each other.

Nor can one inclination perfect one angle: since the quantity also, which is placed between the inclined lines, completes its essence. If then we regard these distinctions, we shall dissolve all absurdities, and discover that the property of an angle is not the collection of a superficies or solid, according to

Apollonius (since these also complete its essence,) but that it is nothing else than a superficies itself, collected into one point, and comprehended by inclined lines, or by one line inclined to itself: and that a solid angle is the collection of superficies mutually inclined to each other. Hence, we shall find that a formed quantum, constituted in a certain relation, supplies its perfect definition. And thus much we have thought requisite to assert concerning the substance of angles, previously contemplating the common essence of every triangle, before we divide it into species.

But since there are three opinions of an angle, Eudemus the Peripatetic, who composed a book concerning an angle, affirms that it is quality. For, considering the origin of an angle, he says that it is nothing else than the fraction of lines: because, if rectitude is quality, fraction also will be quality. And hence, since its generation is in quality, an angle will be entirely quality. But Euclid, and those who call it inclination, place it in the category of relation. But they call it quantity, who say that it is the first interval under a point, that is immediately subsisting after a point. In the number of which is Plutarch, who constrains Apollonius also into the same opinion. For it is requisite (says he) there should be some first interval, under the inclination of containing lines or superficies. But since the interval, which is under a point, is continuous, it is not possible that a first interval can be assumed; since every interval is divisible in infinitum.

Besides, if we any how distinguish a first interval, and through it draw a right line, a triangle is produced, and not one angle. But Carpus Antiochenus says, that an angle is quantity, and is the distance of its comprehending lines, or superficies; and that this is distant by one interval, and yet an angle is not on that account a line: since it is not true that every thing which is distant by only one interval, is a line. But this surely is the most absurd of all, that there should be any magnitude except a line, which is distant only by one interval. And thus much concerning the nature of an angle. But with respect to the division of angles, some consist in superficies, but others in solids. And of those which are in superficies, some are in simple ones, but others in such as are mixt. For an angle may be produced in a cylindric, conic, spherical, and plane superficies.

But of those which consist in simple superficies, some are constituted in the spherical; but others in the plane. For the zodiac itself forms angles, dividing the equinoctal in two parts, at the vertex of the cutting superficies. And angles of this kind subsist in a spherical superficies. But of those which are in planes, some are comprehended by simple lines, others by mixt ones; and others, again, by both. for in the shield-like figure, an angle is comprehended by the axis, and the line of the shield: but one of these lines is mixt, and the other simple. But if a circle cuts a shield, the angle will be comprehended by the circumference, and the ellipsis. And when cissoids, or lines similar to an ivy leaf, closing in one point like the leaves of ivy (from whence they derive their appellation) make an angle, such an angle is comprehended by mixt lines.

Also, when the hippopeda, or line similar to the foot of a mare, which is one of the spirals, inclining to another line, forms an angle, it is comprehended by mixt lines. Lastly, the angles contained by a circumference and a right line, are comprehended by simple lines. But of these again, some are contained by such as are similar in species, but others by such as are dissimilar. For two circumferences, mutually cutting, or touching each other, produce angles: and these triple, for they are either on both sides convex, when the convexities of the circumference are external: or on both sides concave, when both the concavities are external; which they call sistroides; or mixt from convex and concave lines, as the lines called lunulas. But besides this, angles are contained in a twofold manner, by a right line, and a circumference: for they are either contained by a right line and a concave circumference, as the semicircular angle; or by a right line and a convex circumference, as the cornicular angle.

But all those which are comprehended by two right lines, are called rectilinear angles, which have likewise a triple difference. The geometrician, therefore, in the present hypothesis, defines all those angles which are constituted in plane superficies, and gives them the common name of a plane angle. And the genus of these he denominates inclination: but the place, the plane itself, for angles have position: but their origin such, that it is requisite there should be two lines at least, and not three as in a solid. And that these should touch each other, and by touching, must not lie in a right line, as an angle is the inclination and comprehension of lines: but is not distance only, according to one interval. But if we examine this definition, in the first place it appears that it does not admit, an angle can be perfected by one line; though a cissoid, which is but one, perfects an angle.

And, in like manner, the hippopeda. For we call the whole a cissoid, and not its portions (least any one should say, that the conjunction of these forms an angle) and the whole a spiral, but not its parts. Each, therefore, since it is one, forms an angle to itself, and not to another. But after this, he is faulty, in defining an angle to be inclination. For how, on this hypothesis, will there be two angles, from one inclination? How can we call angles equal and unequal? And whatever else is usually objected against this opinion. Thirdly, and lastly, that part of the definition, which says, and not placed in a right line, is superfluous in certain angles, as in those which are formed from orbicular lines. For without the assistance of this part, the definition is perfect; since the inclination of one of the lines to the other, forms the angle. And it is not possible that orbicular angles should be placed in a right line. And thus much we have thought proper to say concerning the definition of Euclid; partly, indeed, interpreting, and partly doubting its truth.

## DEFINITION IX.

But when the Lines containing the Angle, are right, the Angle is called RECTILINEAR.

An angle is the symbol and image of the connection and compression, which subsists in the divine genera, and of that order which collects divisibles into one, partibles into an impartible nature, and the many into conciliating community. For it is the bond of a multitude of lines and superficies, the collector of magnitude into the impartibility of points, and the comprehender of every figure which is composed by its confining nature. On which account, the oracles call the angular junctions of figures, knots, so far as they bring with them an image of connecting union, and divine conjunctions, by which discrete natures mutually cohere with each other. The angles, therefore, subsisting in superficies, express the more immaterial, simple, and perfect unions which superficies contain: but those which are in solids, represent the unions, which proceed even to inferiors, and supply a community to things disjunct, and a construction of the same nature, to things which on every side receive a perfect partition.

But of the angles in superficies, some shadow forth primary and unmixt unions; but others, such as comprehend in themselves, an infinity of progressions. And some, indeed, are the sources of union to intellectual forms; but others, to sensible reasons; and others, again, are copulative of those forms which obtain between these, a middle situation. Hence the angles which are made from circumferences, imitate those causes which envelop intellectual variety in coercive union; for circumferences, hastening to coalesce with each other, are images of intellect, and intellectual forms. On the contrary, rectilineal angles, are the sumbols of those unions which preside over sensibles, and afford a conjunction of the reasons subsisting in these: but mixt angles represent the preservers of the communion, as well of sensible, as of intellectual forms, according to one immoveable union.

It is requisite, therefore, by regarding these paradigms, or exemplars, to render the causes of each. For among the Pythagoreans we shall find various angles dedicated to various gods. Thus, Philolaus, consecrates to some a triangular, but to others a quadrangular angle; and to others, again, different angles. Likewise, he permits the same to many gods, and many to the same god, according to the different powers which they contain. And with a view to this, and to the demiurgic triangle, which is the primary cause of all the ornament of the elements, it appears to me, that Theodorus Asinæus the philosopher, constitutes some of the gods, according to sides; but others, according to angles. The first, indeed, supplying progression and power; but the second, the conjunction of the universe, and the collection of progressive natures again into one.

But these, indeed, direct us to the knowledge of the things which are. And we must not wonder that lines are here said to contain an angle. For the one and impartible nature which is found in these, is adventitious: but in the gods themselves, and in true beings, the whole, and impartible good, precedes things many, and divided.

## DEFINITION X.

When a Right Line standing on a Right Line, makes the successive Angles on each side equal to one another, each of the equal Angles, is a Right Angle; and the instituting Right Line, is called a PERPENDICULAR to that upon which it stands.

## DEFINITION XI.

An OBTUSE ANGLE is that which is greater than a RIGHT ANGLE.

## DEFINITION XII.

But an ACUTE ANGLE, is that which is less than a RIGHT ANGLE.

These are the triple species of angles, which Socrates speaks of in the Republic, and which are received by geometricians from hypothesis; a right-line constituting these angles, according to a division into species; I mean, the right, the obtuse, and the acute. The first of these being defined by equality, identity and similitude; but the others being composed through the nature of the greater and the lesser; and lastly, through inequality and diversity, and through the more and the less, indeterminately assumed. But many geometricians, are unable to render a reason of this division, and use the assertion, that there are three angles, as an hypothesis. So that, when we interrogate them concerning its cause, they answer, this is not to be required of them as geometricians. However, the Pythagoreans, referring the solution of this triple distribution to principles, are not wanting in rendering the causes of this difference of right-lined angles.

For, since one of the principles subsists according to bound, and is the cause of limitation, identity, and equality, and lastly, of the whole of a better co-ordination: but the other is of an infinite nature, and confers on its progeny, a progression to infinity, increase, and decrease, inequality, and diversity of every kind, and entirely presides over the worse series; hence, with great propriety, since the principles of a right-lined angle are constituted by these, the reason proceeding from bound, produces a right angle, one, with respect to the equality of every right angle, endued with similitude, always finite and determinate, ever abiding the same, and neither receiving increment nor decrease. But the reason proceeding from infinity, since it is the second in order, and of a dyadic nature, produces twofold angles about the right angle, distinguished by inequality, according to the nature of the greater and the lesser, and possessing an infinite motion, according to the more and the less, since the one becomes more or less obtuse; but the other more or less acute.

Hence, in consequence of this reason, they ascribe right angles to the pure and immaculate gods of the divine ornaments, and divine powers which proceed into the universe, as the authors of the invariable providence of inferiors; for rectitude, and an inflexibility and immutability to subordinate natures accords with these gods: but they affirm, that the obtuse and acute angles should be ascribed to the gods, who afford progression, and motion, and a variety of powers. Since obtuseness is the image of an expanded progression of forms; but acuteness possesses a similitude to the cause dividing and moving the universe. But likewise, among the things which are, rectitude is, indeed, similar to essence, preserving the same bound of its being; but the obtuse and acute, shadow forth the nature of accidents. For these receive the more and the less, and are indefinitely changed without ceasing.

Hence, with great propriety, they exhort the soul to make her descent into generation, according to this invariable species of the right angle, by not verging to this part more than to that; and by not affecting some things more, and others less. For the distribution of a certain convenience and sympathy of nature, draws it down to material error, and indefinite variety. A perpendicular line is, therefore, the symbol of inflexibility, purity, immaculate, and invariable power, and every thing of this kind. But it is likewise the symbol of divine and intellectual measure: since we measure the altitudes of figures by a perpendicular, and define other rectilineal angles by their relation to a right angle, as by themselves they are indefinite and indeterminate. For they are beheld subsisting in excess and defect, each of which is, by itself, indefinite.

Hence they say, that virtue also stands according to rectitude; but that vice subsists according to the infinity of the obtuse and acute, that it produces excesses and defects, and that the more and the less exhibit its immoderation, and inordinate nature. Of rectilineal angles, therefore, we must establish the right angle, as the image of perfection, and invariable energy, of limitation, intellectual bound, and the like; but the obtuse and acute, as shadowing forth infinite motion, unceasing progression, division, partition and infinitity. As thus much for the theological speculation of angles. But here we must take notice, that the genus is to be added to the definitions of an obtuse and acute angle; for each is right-lined, and the one is greater, but the other less than a right-angle. But it is not absolutely true, that every angle which is less than a right one, is acute.

For the cornicular is less than every right angle, because less than an acute one, yet is not on this account an acute angle. Also, a semi-circular is less than any right-angle, yet is not acute. And the cause of this property is because they are mixt, and not rectilineal angles. Besides, many curve-lined angles appear greater than right-lined angles, yet are not on this account obtuse; because it is requisite that an obtuse should be a right lined angle. Secondly, as it was the intention of Euclid, to define a right-angle, he considers a right line standing upon another right-line, and making the angles on each side equal. But he defines an obtuse and acute angle, not from the inclination of a right line to either part, but from their relation to a right-angle. For this is the measure of angles deviating from the right, in the same manner as equality of things unequal.

But lines inclined to either part, are innumerable, and not one alone, like a perpendicular. But after this, when he says, (the angles equal to one another) he exhibits to us a specimen of the greatest geometrical diligence; since it is possible that angles may be equal to others, without being right. But when they are equal to one another, it is necessary they should be right. Besides, the word successive appears to me not to be added superfluously, as some have improperly considered it; since it exhibits the reason of rectitude. For it is on this account that each of the angles is right; because, when they are successive, they are equal. And, indeed, the insisting right-line, on account of its inflexibility to either part, is the cause of equality to both, and of rectitude to each. The cause, therefore, of the rectitude of angles, is not absolutely mutual equality, but position in a consequent order, together with equality.

But, besides all this, I think it here necessary to call to mind, the purpose of our author; I mean, that he discourses in this place, concerning the angles consisting in one plane. And hence, this definition is not of every perpendicular; but of that which is in one and the same plane. For it is not his present design to define a solid angle. As, therefore, he defines, in this place, a plane angle, so likewise a perpendicular of this kind. Because a solid perpendicular ought not to make right angles to one right-line only; but to all which touch it, and are contained in its subject plane: for this is its necessary peculiarity.

## DEFINITION XIII.

A bound is that which is the Extremity of any thing

A Bound, in this place, is not to be referred to all magnitudes, for there is a bound and extremity of a line; but to the spaces which are contained in superficies, and to solid bodies. For he now calls a bound, the ambit which terminates and distinguishes every space. And a bound of this kind, he defines to be an extremity: but not after the manner in which a point is called the extremity of a line, but according to its property of including and excluding from circumjacent figures. But this name is proper to geometry in its infant state, by which they measured fields, and preserved their boundaries distinct and without confusion, and from which they arrived at the knowledge of the present science. Since, therefore, Euclid calls the external ambit, a bound, it is not without propriety that he, by this means, defines the extremity of spaces. For by this, every thing comprehended is circumscribed. I say, for example, in a circle, its bound and extremity is the circumference; but itself, a certain plane space: and so of the rest.

## DEFINITION XIV.

A figure is that which is comprehended by one or more Boundaries.

Because figure is predicated in various ways, and is divided into different species, it is requisite, in the first place, to behold its differences; and afterwards to discourse concerning that figure which is proposed in this Definition. There is, then, a certain figure which is constituted by mutation, and is produced from passion, while the recipients of the figure are disturbed, divided, or taken away; while they receive additions, or are altered, or suffer other various affectations. There is also figure, which is produced by the potter's, or statuary's art, according to the pre-existent reason, which art itself contains: art, indeed, producing the form, but matter receiving from thence, form, and beauty, and elegance. But there are still more noble and more illustrious figures than these, the skilful operations of nature. Some, indeed, existing in the elements under the moon, and having a power of comprehending the reasons those elements contain: but others are situated in the celestial regions, distinguishing their powers, and endless revolutions.

For the heavenly bodies, both when considered by themselves, and with relation to each other, exhibit an abundant and admirable variety of figures; and at different times they present to our view different forms, bringing with them a splendid image of intellectual species; and, by their elegant and harmonious revolutions, describing the incorporeal and immaterial powers of figures. But there are, again, besides all these, most pure and perfect beauties, the figures of souls, which, because they are full of life, and self-motive, have an existence prior to things moved by another; and which, because they subsist immaterially, and without any dimension, excel the forms which are endued with dimension and matter. In the nature of which we are instructed by Timæus, who has explained to us the demiurgic, and essential figure of souls. But again, the figures of the intellects are by far more divine than the figures of souls; for these, on every side, excel partible essences; are every where resplendent with impartible and intellectual light; are prolific, effective, and perfective of the universe; are equally present, and firmly abide in all things; and procure union to the figures of souls; but recall the mutation of sensible figures to the limitation of their proper bound.

Lastly, there are, separate from all these, those perfect, uniform, unknown, and ineffable figures of the gods, which are resident, indeed, in the figures of intellects; but jointly terminate all figures, and comprehend all things in their unifying boundaries. The properties of which the theurgic art, also expressing, surrounds various resemblances of the gods, with various figures. And some, indeed, it fashions by characters, in an ineffable manner; for characters of this kind, manifest the unknown powers of the gods: but others it imitates by forms and images; fashioning some of them erect, and others sitting; and some similar to a heart, but others spherical, and other expressed by different figures. And again, some it fabricates of a simple form; but others it composes from a multitude of forms; and some are sacred and venerable; but others are domestic, exhibiting the peculiar gentleness of the gods.

And some it constructs of a severe aspect; and lastly, attributes to others, different symbols, according to the similitude and sympathy pertaining to the gods. Since, therefore, figure derives its origin from the gods themselves, it arrives, by a gradual progression, even to inferiors, in these also appearing from primary causes. Since it is requisite to suppose the perfect before the imperfect, and things situated in the stability of their own essence, prior to those which subsist in others, and previous to things full of their own privation, such as preserve their proper nature sincere. Such figures, therefore, as are material, participate of material inelegance, and do not possess a purity convenient to their nature. But the celestial figures are divisible, and subsist in others. And the figures of souls are endued with division, and variety, and involution of every kind; but the figures of intellects, together with immaterial union, possess a progression into multitude.

And lastly, the figures of the gods are free, uniform, simple, and generative; they subsist before all things, containing all perfection in themselves, and extending from themselves to all things, the completion of forms. We must not, therefore, listen to, and endure the opinions of many, who affirm, that certain additions, ablations, and alterations, produce sensible figures, (for motions, since they are imperfect, cannot possess the principle and primary cause of effects; nor could the same figures often be produced from contrary motions; for the same form is sometimes generated from addition and detraction,) but we must consider operations of this kind as subservient to other purposes in generation, and derive the perfection of figure from other primogenial causes. Nor must we subscribe to their opinion, who assert that figures destitute of matter can have no subsistence, but those only which appear in matter.

Nor to theirs, who acknowledge, indeed, that they are external to matter, but consider them as subsisting alone, according to thought and abstraction. For where shall we preserve in safety, the certainty, beauty, and order of figures, among things which subsist by abstraction? For, since they are of the same kind with sensibles, they are far distant from indubitable and pure certainty. But from whence do they derive the certainty, order, and perfection which they receive? For they either derive it from sensibles (but they have no subsistence in these), or from intelligibles (but in these they are more perfect), since, to say from that which is not, is the most absurd of all. For nature does not produce imperfect figures, and leave the perfect without any subsistence. Nor is it lawful, that our soul should fabricate more certain, perfect, and orderly figures, than intellect and the gods themselves.

There are, therefore, prior to sensible figures, self-moving, intellectual, and divine reasons of figures. And we are excited, indeed, from the obscurity of sensible forms, but we produce internal reasons, which are the lucid images of others. And we posses a knowledge of sensible figures, by their exemplars resident in soul, but we comprehend by images, such as are intellectual and divine. For the reasons we contain, emerging from the dark night of oblivion, and propagating themselves in sciential variety, exhibit the forms of the gods, and the uniform bounds of the universe, by which they ineffably convert all things into themselves. In the gods, therefore, there is both an egregious knowledge of universal figures, and a power of generating and constituting all inferiors. But in natures, figures are endued with a power generative of apparent forms; but are destitute of cognition and intellectual perception.

And, in particular souls, there is, indeed, an immaterial intellection, and a self-energizing knowledge; but there is wanting a prolific, and efficacious cause. As, therefore, nature, by her forming power presides over sensible figures, in the same manner, soul, by her gnostic energy, drops in the phantasy as in a mirror, the reasons of figures. But the phantasy receiving these in her shadowy forms, and possessing images of the inherent reasons of the soul, affords by these the means of inward conversion to the soul, and of an energy directed to herself, from the spectres of imagination. Just as if any one beholding his image in a mirror, and admiring the power of nature, and his own beauty, should desire to see himself in perfection, and should receive a power of becoming, at the same time, the perceiver, and the thing perceived. For the soul, after this manner, looking abroad into the bright mirror of the phantasy, and surveying the shadowy figures it contains, and admiring their beauty and order, pursues, in consequence of her admiration, the reasons from which these images proceed; and being wonderfully delighted, dismisses their beauty, as conversant about spectres alone; but afterwards seeks her own purer beauty, and desires to pass into her own profound retreats, and there to perceive the circle and the triangle, and all things subsisting together, in an impartible manner, and to insert herself in the objects, to contract her multitude into one; and lastly, to behold the occult and ineffable figures of the gods, seated in the most sacred and divine recesses of her nature.

She is likewise desirous of bringing into light, from its awful concealment, the solitary beauty of the gods, and of perceiving the circle, subsisting in its true perfection, more impartible than any centre, and the triangle without interval; and lastly, by ascending into an union with herself, of surveying every object which is subject to the power of cognition. The figure, therefore, which is self-motive, precedes that which is moved by another; and the impartible that which is self-motive: but that which is the same with one, precedes the impartible itself. For all things are bounded, when they return to the unities of their nature; since all things pass through these as a divine entrance into being. And thus much for this long digression, which we have delivered according to the sentiments of the Pythagoreans. But the geometrician, contemplating that figure which is seated in the phantasy, and defining this, in the first place, (since this definition agrees with sensibles, in the second place) says, that figure is that which is comprehended by one or more boundaries.

For, since he receives it together with matter, and conceives of it as distant with intervals, he does not improperly call it finite and terminated. [Since every thing which contains either intelligible or sensible matter, is allotted an adventitious bound; and is not itself bound, but that which is bounded.] Nor is it the bound of itself; but one of its powers is terminating, and the other terminated. Nor does it subsist in bound itself, but is contained by bound. For figure is joined to quantity, and subsists together with it; and, at the same time, quantity is subjected to figure; but the reason and aspect of that quantity is nothing else than figure and form. Since, indeed, reason terminates quantity, and adds to it a particular character and bound, either simple or composite. For, since this also exhibits the two-fold progression of bound and infinite in its proper forms, (in the same manner as the reason of an angle,) it invests the objects of its comprehension with one boundary and simple form, according to bound, but with many, according to infinity.

Hence, every thing figured, vindicates to itself either one boundary, or a many. Euclid, therefore, denominating that which is figured and material, and annexed to quantity figure, does not improperly say, that it is contained by one or more terms. But Possidonius defines figure to be concluding bound, separating the reason of figure from quantity; and considering it as the cause of terminating, defining, and comprehending quantity. For that which encloses, is different from that which is enclosed; and bound from that which is bounded. And Possidonius, indeed, seems to regard the external surrounding bound; but Euclid, the whole subject. Hence, the one calls the circle a figure, with relation to its whole plane, and exterior ambit; but the other with relation to its circumference only. And the one defines that which is figured, and which is beheld together with its subject: but the other desires to define the reason of the circle; I mean that which terminates and concludes its quantity.

But if any logician, and captious person, should blame the definition of Euclid, because he defines genus from species (for things contained by one or more terms, are the species of figure,) we shall assert, in opposition to such an objection, that genera also pre-occupy in themselves the powers of species. And when men of ancient authority, were willing to manifest genera themselves, from those powers which genera contain, they appeared, indeed, to enter on their design from species, but, in reality, they explained genera from themselves, and from the powers which they contain. The reason of figure, therefore, since it is one, comprehends the differences of many figures, according to the bound and infinity residing in its nature. And he who defined this reason, was not void of understanding, whilst he comprehended in a definition, the differences of the powers it contained.

But you will ask, From whence does the reason of figure originate, and by what causes is it perfected? I answer, that it first arises from bound and infinite, and that which is mixed from these. Hence it produces some species from bound, others from infinite, and others from the mixt. And this it accomplishes by bringing the form of bound to circles; but that of the infinite, to right-lines: and that of the mixt to figures composed from right and circular lines. But, in the second place, this reason is perfected from that totality which is separated into dissimilar parts. From whence, indeed, it occasions a whole to every form, and each figure is cut into different species. For a circle, and every right-lined figure may be divided, by reason or proportion, into dissimilar figures; which is the business of Euclid in his book of divisions, where he divides one figure into figures similar to such as are given; but another into such as are dissimilar.

In the third place, it is invigorated from accumulated multitude, and, on account of this, extends forms of very kind, and produces the multiform reasons of figures. Hence, in propagating itself, it does not cease till it arrives at something last, and has unfolded all the variety of forms. And, as in the intelligible world, one is shewn to abide in that which is; and, at the same time, that which is in one, so likewise, reason exhibits circular in right-lined figures; and, on the contrary, rectilinear comprehended in circular figures. And it peculiarly manifests its whole nature in each, and all these in all. Since the whole subsists in all collectively, and in each separate and apart. From that order, therefore, it is endued with this power. In the fourth place, it receives from the first of numbers, the measures of the progression of forms.

From whence it constitutes all figures according to numbers; some, indeed, according to the more simple, but others according to the more composite.

For triangles, quadrangles, quinquangles, and all multangles, proceed in infinitum, together with the mutations of numbers. But the cause of this is, indeed, unknown to the vulgar, though, to those who understand where number and figure subsist, the reason is manifest. Fifthly, it is replete with that division of forms, which divides forms into other similar forms, from another second totality, which is also distributed into similar parts. And by this, a triangular reason is divided into triangles, and a quadrangular reason into quadrangles. And hence, exercising our inward powers, we effect what I have said in images, since it pre-existed by far the first in its principles. But by regarding these distributions, we may render many causes of figures, reducing them to their first principles.

And the more common, or geometrical figure, is allotted an order of this kind, and from so many causes, receives the perfection of its nature. But, from hence it advances to the genera of the gods, and is variously attributed according to its various forms, and energizes differently in different gods. to some, indeed, affording more simple figures; but to others, such as are more composite. And to some, again, assigning primary figures, and those which are produced in superficies; but to others (entering the tumor of solid bodies) such figures, as in solids are convenient to themselves. For all figures, indeed, subsist in all, since the forms of the gods are accumulated, and full of universal powers: but, by their peculiarity, they produce one thing according to another. For one possesses all things circularly, another in a triangular manner, but another according to a quadrangular reason. And in a similar manner in solids.

## DEFINITION XV.

A circle is a Plane Figure, comprehended by one Line, which is called the Circumference, to which all Right Lines falling from a certain Point within the Figure, are equal to each other.

## DEFINITION XVI.

And that point is called the CENTRE of the CIRCLE.

division into the heavens, and the universal regions of generation, you must assign to the heavens a circular form; but to generation, that of a right line. For whatever among generable natures is circular, descends from the heavens;

since generation revolves into itself, through their circumvolutions, and reduces its unstable mutation to a regular and orderly continuance. But if you distribute incorporeal natures into soul and intellect, you will say, that the circle belongs to the intellect, and the right line to the soul. And on this account, the soul, by its conversion to intellect, is said to be circularly moved; and it possesses the same proportion to intellect, as generation to the heavens. For it is circularly moved, (says Socrates,) because it imitates intellect. But the generation and progression of soul is made according to a right-line.

For it is the property of the soul to apply herself at different times to different forms. But if you wish to divide into body and soul, you must constitute every thing corporeal, according to the right line; but you must assign to every animal a participation of the identity and similitude of the circle. For body is a composite, and is endued with various powers, similar to right-lined figures: but soul is simple and intelligent; self-motive, and self-operative; converted into, and energizing in herself. From whence, indeed, Timæus also, when he had composed the elements of the universe from right-lined figures, assigned to them a circular motion and formation, from that divine soul which is seated in the bosom of the world. And thus, that the circle every where holds the first rank, in respect of other figures, is sufficiently evident from the preceding observations.

But it is requisite to survey its whole series, beginning supernally, ending in inferiors, and perfecting all things, according to the aptitude of the natures which receive its alliance. To the gods, therefore, it affords a conversion to their causes, and ineffable union: it occasions their abiding in themselves, prevents their departing from their own beatitude, strengthens their highest unions, as centres desirable to inferior natures, and stably places about these the multitude of the powers which the gods possess, containing them in the simplicity of their essences. But the circle affords to intellectual natures, a perpetual energy in themselves, is the cause of their being filled with knowledge from themselves, and of possessing in their essences, intelligibles contractedly; and of perfecting intellections in themselves. For every intellect, proposes to itself that which is intelligible; and this is a centre to intellect, about which it continually revolves: for intellect folds itself, and operates about this, and is united within itself on all sides, by universal intellectual energies.

But it extends to souls by illumination, a self-vital, and self-motive power, and an ability of turning, and leaping round intellect, and of returning according to proper convolutions, unfolding the impartiality of intellect. Again, the intellectual orders excel souls after the manner of centres, but souls energize circularly about their nature. For every soul, according to its intellectual part, and the supreme one, which is the very flower of its essence, receives a centre: but, according to its multitude, it has a circular revolution, desiring, by this means, to embrace the intellect which it participates. But, to the celestial bodies, the circle affords an assimilation to intellect, equality, a comprehension of the universe, in proper limits, revolutions which take place in determinate measures, a perpetual subsistence, a nature without beginning and end, and every thing of this kind.

And to the elements under the concave of the moon's orb, it is the cause of a period, conversant with mutations; and assimilation to the heavens; that which is without generation, in generated natures; that which abides in things which are moved; and whatever is bounded in partible essences. For all things are perpetual, through the circle of generation; and equability is every where preserved on account of the reciprocation of corruption. Since, if generation did not return, in a circular revolution, in a short space of time, the order, and all the ornament of the elements would vanish. But again, the circle procures to animals and plants, that similitude which is found in generations: for these are produced from seeds, and seeds from these. Hence, generation here, and a circumvolution, alternately takes place, from the imperfect to the perfect, and the contrary; so that corruption subsists together with generation.

But, besides this, to unnatural productions it imposes order, and reduces their indeterminate variety to the limitation of bound; and, through this, nature herself is gracefully ornamented in the last vestiges of her powers. Hence, things contrary to nature have a revolution according to determinate numbers, and not only fertility, but also sterility, subsists according to the alternate convolutions of circles (as the discourse of the Muses evinces), and all evils though they are dismissed from the presence of the gods, into the place of mortals, yet these roll round, says Socrates, and to these there is present a circular revolution, and a circular order; so that nothing immoderate and evil is deserted by the gods; but that providence, which is perfective of the universe, reduces also the infinite variety of evils, to bound, and an order convenient to their nature.

The circle, therefore, is the cause of ornament to all things, even to the last participations, and leaves nothing destitute of itself, since it supplies beauty, similitude, formation, and perfection to the universe. Hence too, in numbers, it contains the middle centres of the whole progression of numbers, which revolves from unity to the decad (or ten). For five and six exhibit a circular power, because, in the progressions from themselves, they return again into themselves, as is evident in the multiplication of these numbers. Multiplication, therefore, is an image of progression, since it is extended into multitude; but an ending in the same species, is an image of regression into themselves. But a circular power affords each of these, exciting, indeed, as from an abiding centre, those causes which are productive of multitude; but converting multitude after the productions to their causes.

Two numbers, therefore, having the properties of a circle, possess the middle place between all numbers: of which one, indeed, precedes every convertible genus of males and an odd nature; but the other, recalls every thing feminine and even, and all prolific series, to their proper principles, according to a circular power. And thus much concerning the perfection of the circle. Let us now contemplate the mathematical definition of the circle, which is every way perfect. In the first place, therefore, he defines it a figure, because, indeed, it is finite, and every where comprehended by one limit, and is not of an infinite nature, but associated to bound. Likewise plane, because, since figures are either beheld in superficies, or in solid bodies, a circle is the first of plane figures, excelling solids in simplicity, but possessing the proportion of unity to planes.

But comprehended by one line, because it is similar to one, by which it is defined, and because it does not extrinsically receive a variety of surrounding terms. And again, that this line makes all the lines drawn to it from a certain point within equal, because of the figures which are bounded by one line, some have all the lines proceeding from the middle equal; but others not at all. For the ellipsis is comprehended by one line, yet all the lines issuing from the centre, and bounded by its curvature, are not equal, but only two. Also the plane, which is included by the line called a cissoid, has one containing line, yet it does not contain a centre, from which all the lines are equal. But, because the centre in a circle is entirely one point (for there are not many centres of one circle), on this account, the geometrician adds, that line falling from one point to the bound of the circle are equal.

For there are infinite points within it, but of all these, one only has the power of a centre. And because this one point, from which all the lines drawn to the circumference of the circle are equal, is either within the circle, or without (for every circle has a pole, from which all the lines are drawn to its circumference are equal), on this account he adds, of the points within the figure, because, here he receives the centre alone, and not the pole. For he wishes to behold all its properties in one plane, but the pole is more elevated than the subject plane. Hence, he necessarily adds, in the end of the definition, that this point, which is placed within the circle, and to which all right lines drawn from it to the circumference, are equal, is the centre of the circle. For there are only two points of this kind, the pole and the centre.

But the former is without, and the other within the plane. Thus, for instance, if you conceive a perpendicular standing on the centre of a circle, its superior extremity is the pole: for all lines drawn from it to the circumference of the circle, are demonstrated to be equal. And, in like manner, in a cone, the vertex of the cone, is the pole of the circle at the base. And thus far we have determined what a circle is, and its centre, and what the nature is of its circumference, and the whole circular figure. Again, therefore, from these, let us return to the speculation of their exemplars, contemplating in them the centre, according to one impartible and stable excellence. But the distances from the centre, according to the progressions which are made from one, to multitude infinite in capacity. And the circumference of the circle, according to the regression of the progressions to the centre, by means of which the multitude of powers are rolled round their union, and all of them hasten to its comprehension, and desire to energize about its indivisible embrace.

And, as in the circle itself, all things subsist together, the centre, intervals, and external circumference; so in these which are its image, one things has not an essence pre-existent, and another consequent in time; but all things are, indeed, together, permanency, progression, and regression. But these differ from those, because the former subsist indivisibly, and without any dimension; but the latter with dimension, and in a divisible manner; the centre existing in one place, the lines emanating from the centre, in another; and the external circumference terminating the circle, having a still different situation.

But there all things abide in one: for if you regard that which performs the office of a centre, you will find it the receptacle of all things. If the progression distant from the centre, in this, likewise, you will find all things contained.

And, in a similar manner, if you regard its regression. When, therefore, you are able to perceive all things subsisting together, and have taken away the defect proceeding from dimension, and have removed from your inward vision, the position about which partition subsists, you will find the true circle, advancing to itself, bounding, and energizing in itself, existing both one and many, and abiding, proceeding, and returning; likewise firmly establishing that part of its essence which is most impartible, and especially singular; but advancing from this according to rectitude, and the infinity which it contains; and rolling itself from itself to one, and exciting itself by similitude and identity to the impartible centre of its nature, and to the occult power of the one which it contains. But this one, which the circle contains, and environs in its bosom, it emulates according to the multitude of its own nature.

For that which is convolved, imitates that which abides, and the periphery is as a centre which is distant with interval, and nods to itself, hastening to receive, and to become one with the centre, and to terminate its regress where it received the principle of its progression. For the centre is every where in the place of that which is lovely, and the object of desire, presiding over all things which subsist about its nature, and existing as the beginning and author of all progressions. And this the mathematical centre also expresses, by terminating all the lines falling from itself to the circumference, and by affording to them equality, as an image of proper union. But the oracles likewise define the centre, after this manner: The centre is that from which and to which all the lines to the circumference are equal. Indicating the beginning of the distance of the lines, by the particle from which; but the middle of the circumference by the particle to which: for this, in every part, is joined with the centre.

But if it be necessary to declare the first cause, through which a circular figure appears and receives its perfection, I affirm, that it is the supreme order of intelligibles. For the centre, indeed, is assimilated to the cause of bound; but the lines emanating from this, and which are infinite, with respect to themselves, both in multitude and magnitude, represent infinity; and the line which terminates their extension, and conjoins the circular figure with the centre, is similar to that occult ornament, consisting from the intelligible orders; which Orpheus also says, is circularly borne, in the following words, But it is carried with an unwearied energy, according to an infinite circle. For, since it is moved intelligibly, about that which is intelligible, having it for the centre of its motion, it is, with great propriety, said to energize in a circular manner.

Hence, from this also, the triadic god proceeds, who contains in himself the cause of the progression of right-lined figures. For on this account, wise men, and the most mystic of theologists have fabricated his name. [Hence too, it is manifest that a circle is the first of all figures:] but a triangle is the first of such as are right-lined. Figures, therefore, appear first in the regular ornaments of the gods; but they have a latent subsistence, according to pre-existent causes, in intelligible essences.

## DEFINITION XVII.

A diameter of a Circle, is a certain straight Line, drawn through the Centre, which is terminated both ways by the Circumference of the Circle, and divides the Circle into two equal Parts.

Euclid here perspicuously shews, that he does not define every diameter, but that which belongs to a circle only. Because there is a diameter of quadrangles and all parallelograms, and likewise of a sphere among solid figures. But in the first of these, it is denominated a diagonal: but in a sphere, the axis; and in circles the diameter only. Indeed, we are accustomed to speak of the axis of an ellipsis, cylinder, and cone; but of a circle, with propriety, the diameter. This, therefore, in its genus, is a right-line; but as there are many right-lines in a circle, as likewise infinite points, one of which is a centre, so this only is called a diameter, which passes through the centre, and neither falls within the circumference, nor transcends its boundary; but is both ways terminated by its comprehensive bound. And these observations exhibit its origin.

But that which is added in the end, that it also divides the circle into two equal parts, indicates its proper energy in the circle, exclusive of all other lines drawn through the centre, which are not terminated both ways by the circumference. But they report, that Thales first demonstrated, that the circle was bisected by the diameter. And the cause of this bisection, is the indeclinable transit of the right line, through the centre. For, since it is drawn through the middle, and always preserves the same inflexible motion, according to all its parts, it cuts off equal portions on both sides to the circumference of the circle. But if you desire to exhibit the same mathematically, conceive the diameter drawn, and one part of the circle placed on the other. Then, if it is not equal, it either falls within, or without, but the consequence either of these ways must be, that a less right-line will be equal to a greater.

Since all lines from the centre to the circumference are equal. The line, therefore, which tends to the exterior circumference, will be equal to that which tends to the interior. But this is impossible. These parts of the circle, then, agree, and are on this account equal. But here a doubt arises, if two semicircles are produced by one diameter, and infinite diameters may be drawn through the centre, a double of infinities will take place, according to number. For this is objected by some against the section of magnitudes to infinity. But this we may solve by affirming, that magnitude may, indeed, be divided infinitely, but not into infinites. For this latter mode produces infinites in energy, but the former in capacity only. And the one affords essence to infinite, but the other is the source of its origin alone. Two semi-circles, therefore, subsist together with one diameter, yet there will never be infinite diameters, although they may be infinitely assumed.

Hence, there can never be doubles of infinites; but the doubles which are continually produced, are the doubles of finites; for the diameters which are always assumed, are finite in number. And what reason can be assigned why every magnitude should not have finite divisions, since number is prior to magnitudes, defines all their sections, pre-occupies infinity, and always determines the parts which rise into energy from dormant capacity?

## DEFINITION XVIII.

A semicircle is the Figure contained by the Diameter, and that Part of the Circumference which is cut off by the Diameter.

## DEFINITION XIX.

But the CENTRE of the Semi-circle, is the same with that of the Circle.

From the definition of a circle Euclid finds out the nature of the centre, different from all the other points which the circle contains. But from the centre he defines the diameter, and separates it from the other right lines, which are described within the circle. And from the diameter, he teaches the nature of the semi-circle; and informs us that it is contained by two terms, always differing from each other, viz. a right-line and a circumference: and that this right-line is not any one indifferently, but the diameter of the circle. For both a less and a greater segment of a circle, are contained by a rightline and circumference; yet these are not semi-circles, because the division of the circle is not made through the centre. All these figures, therefore, are biformed, as a circle was monadic, and are composed from dissimilars. For every figure which is comprehended by two terms, is either contained by two circumferences, as the lunular: or by a right-line and circumference, as the above mentioned figures; or by two mixt lines, as if two ellipses intersect each other (since they enclose a figure, which is intercepted between them), or by a mixt line and circumference, as when a circle cuts an ellipsis; or by a mixt and right-line, as the half of an ellipsis. But a semi-circle is composed from dissimilar lines, yet such as are, at the same time, simple, and touching each other by apposition. Hence, before he defines triadic figures, he, with great propriety, passes from the circle to a biformed figure. For two right-lines can, indeed, never comprehend space. But this may be effected by a right-line and circumference. Likewise by two circumferences, either making angles, as in the lunular figure; or forming a figure without angles, as that which is comprehended by concentric circles. For the middle space intercepted between both, is comprehended by two circumferences; one interior, but the other exterior, and no angle is produced. For they do not mutually intersect, as in the lunular figure, and that which is on both sides convex. But that the centre of the semi-circle is the same with that of the circle, is manifest. For the diameter, containing in itself the centre, completes the semi-circle, and from this all lines drawn to the semi-circumference are equal. For this is a part of the circumference of the circle. But equal right lines proceed from the centre to all parts of the circumference. The centre, therefore, of the circle and semi-circle is one and the same. And it must be observed, that among all figures, this alone contains the centre in its own perimeter, I say, among all plane figures. Hence you may collect, that the centre has three places.

For it is either within a figure, as in the circle; or in its perimeter, as in the semi-circle; or without the figure, as in certain conic lines. What then is indicated by the semi-circles, having the same centre with the circle, or of what things does it bear an image, unless that all figures which do not entirely depart from such as are first, but participate them after a manner, may be concentric with them, and participate of the same causes? For the semicircle communicates with the circle doubly, as well according to the diameter, as according to the circumference. On this account, they posses a centre also in common. And perhaps, after the most simple principles, the semi-circle is assimilated to the second co-ordinations, which participate those principles; and by their relation to them, although imperfectly, and by halves, they are, nevertheless, reduced to that which is, and to their first original cause.

## DEFINITION XX.

Rectilinear figures are those which are comprehended by Straight Lines.

## DEFINITION XXI.

Trilateral figures, or TRIANGLES, by three Straight Lines.

## DEFINITION XXII.

QUADRILATERAL, by four Straight Lines.

## DEFINITION XXIII.

Multilateral figures, or POLYGONS, by more than four Straight Lines.

After the monadic figure having the relation of a principle to all figures, and the biformed semi-circle, the progression of right-lined figures in infinitum, according to numbers, is delivered. For on this account also, mention was made of the semi-circle, as communicating according to terms or boundaries; partly, indeed, with the circle, but partly with right-lines: just as the duad is the medium between unity and number. For unity, by composition, produces more than by multiplication; but number, on the contrary, is more increased by multiplication than composition: and the duad, whether multiplied into, or compounded with itself, produces an equal quantity. As, therefore, the duad is the middle of unity and number, for likewise, a semicircle communicates, according to its base, with right-lines; but according to its circumference, with the circle.

But right-lined figures proceed orderly to infinity, attended by number and its bounding power, which begins from the triad. On this account, Euclid also begins from hence. For he says, trilateral and quadrilateral, and the following figures, called by the common name of multilateral: since trilateral figures are also multilaterals; but they have likewise a proper, besides a common denomination. But, as we are but little able to pursue the rest, on account of the infinite progression of numbers, we must be content with a common denomination. But he only makes mention of trilaterals and quadrilaterals, because the triad and tetrad are the first in the order of numbers; the former being a pure odd among the odd; but the latter, an entire even among even numbers. Euclid, therefore, assumes both in the origin of right-lined figures, for the purpose of exhibiting their subsistence, according to all even and odd numbers.

Besides, since he is about to teach concerning these in the first book, as especially elementary (I mean triangles and parallelograms) he does not undeservedly, as far as to these, establish a proper enumeration: but he embraces all other right-lined figures by a common name, calling them multilaterials: but of these enough. Again, assuming a more elevated exordium, we must say, that of plane figures, some are contained by simple lines, others by such as are mixt, but others again by both. And of those which are comprehended by simple lines, some are contained by similars in species, as right-lines; but others by dissimilars in species, as semi-circles, and segments, and apsides, which are less than semi-circles. Likewise of those which are contained by similars in species, some are comprehended by a circular line; but others by a right-line.

And of those comprehended by a circular line, some are contained by one, others by two, but others by more than two. By one, indeed, the circle itself. But by two, some without angles, as the crowns terminated by concentric circles; but others angular as the lunula. And of those comprehended by more than two, there is an infinite procession. For there are certain figures contained by three and four and succeeding circumferences. Thus, if three circles touch each other, they will intercept a certain trilateral space; but if four, one terminated by four circumferences, and in like manner, by a successive progression. But of those contained by right lines, some are comprehended by three, others by four, and others by a multitude of lines. For neither is space comprehended by two right-lines, nor much more by one right-line. Hence, every space comprehended by one boundary, or by two, is either mixt or circular.

And it is mixt in a twofold manner, either because the mixt lines comprehend it, as the space intercepted by the cissoidal line; or because it is contained by lines dissimilar in species, as the apsis: since mingling is two-fold, either by apposition or consfusion. Every right-lined figure, therefore, is either trilateral, or quadrilateral, or gradually multilateral; but every trilateral, or quadrilateral, or multilateral figure, is not right-lined; since so great a number of sides is also produced from circumferences. And thus much concerning the division of plane figures. But we have already asserted, that rectitude of progression is both a symbol of motion and infinity, and that it is peculiar to the generative co-ordinations of the gods, and to the producers of difference, and to the authors of mutation and motion. Right-lined figures, therefore, are peculiar to these gods, who are the principles of the prolific energy of the whole progression of forms. On which account, generation also, was principally adorned by these figures, and is allotted its essence from these, so far as it subsists in continual motion and mutation without end.

## DEFINITION XXIV.

Of three-sided FIGURES, an Equilateral Triangle is that which has three equal Sides.

## DEFINITION XXV.

An ISOSCELES TRIANGLE, is that which has only two Sides equal.

## DEFINITION XXVI.

A scalene Triangle, is that which has three unequal Sides.

## DEFINITION XXVII.

Arightangled Triangle is that which has a Right Angle.

## DEFINITION XXVIII.

An OBTUSE-ANGLED TRIANGLE is that which has an Obtuse Angle.

## DEFINITION XXIX.

An ACUTE-ANGLED TRIANGLE is that which has three Acute Angles.

The division of triangles sometimes commences from angles, but sometimes from sides. And that, indeed, which originates from sides, precedes as known; but that from angles follows as a proper distribution. For these three angles alone belong to right-lined figures, viz. the right, the obtuse, and the acute: but the equality and inequality of sides subsist also in non-rectilinear figures. Euclid says, therefore, that of triangles, some are equilateral, others isosceles, and others scalene: for they have either all their sides equal, or all unequal, or only two equal. And again, that of triangles some are right-angled, others obtuse-angled, and others acute-angled. And he defines a right-angled triangle, that which has one right angle, as likewise an obtuseangled triangle, that which has one obtuse angle: for it is impossible that a triangle can have more than one right, or obtuse angle.

But he defines an acute-angled triangle, that which has all its angles acute. For here it was not sufficient that it should have only one acute; since, in this case, all triangles would be acute-angled, as every triangle has necessarily two acute angles. But, to possess three acute angles, is the property of an acute-angled triangle alone. But Euclid appears to me to have made a separate division into angles and sides, from considering this alone, that every triangle is not also trilateral. For there are quadrilateral triangles, which are called by mathematicians themselves that is, similar to the point of a spear: but by Zenodorus that is, having a hollow angle. For on one of the sides of a trilateral figure, constitute two right-lines inwardly; by this means a certain space will be enclosed, which is comprehended by external and internal right-lines, and which has three angles; one, indeed, contained by the external lines; but two comprehended by these and the internal lines, at the extremities in which these lines are conjoined.

A figure of this kind, therefore, is a quadrilateral triangle. And hence, it does not immediately follow, that because a figure has three angles (whether they are all acute, or one right, or one obtuse), we shall find it trilateral; for it may be, perhaps, quadrilateral. In like manner, you may also find quadrangles having more than four sides. And therefore, we must not rashly determine the number of sides from the multitude of angles. But of this enough. But the Pythagoreans affirm that the triangle is simply the principle of generation, and of the formation of generable natures. On which account, Timæus says, that natural reasons, as well as those of the construction of the elements, are triangular. For they are distant by a triple interval, are on all sides collective of partible, and variously mutable natures, are replete with material infinity, and bear before themselves the conjunctions of material bodies, loosened and free: as, indeed, triangle also are comprehended by three right-lines, but they possess angles which collect the multitude of lines, and afford to them an adventitious angle and conjunction.

With great propriety, therefore. Philolaus has consecrated the angle of a triangle to four gods, Saturn, Pluto, Mars and Bacchus, comprehending in these the whole quadripartite ornament of the elements descending from the heavens, or from the four segments of the zodiac. For Saturn constitutes an essence wholly humid and frigid; but Mars a nature totally fiery; and Pluto contains the whole terrestial life; but Bacchus governs a humid and hot generation; of which wine also is a symbol, for this is humid and hot. Hence, all these gods differ according to their operations in inferior concerns: but they are mutually united according to their proper natures. And on this account, Philolaus collects their union according to one angle. But if the differences of triangles contribute to generation, we shall very properly confess that a triangle is the principle and author of the constitution of sublunary natures.

For a right angle, indeed, affords them essence, and determines the measure of being; and the reason of a right-angled triangle produces the essence of the elements of generable natures; but an obtuse angle assigns to them universal distance; and the reason of an obtuse-angled triangles increases material forms in magnitude, and in mutation of every kind. But an acute angle effects their divisible nature; and the reason of an acute-angled triangle prepares them to receive infinite division. But, simply, a triangular reason constitutes the essence of material bodies distant with interval, and on all sides divisible. And thus much should we speculate concerning the nature of triangles. But from these divisions you may understand, that all the species of triangles are neither more nor less than seven. For the equilateral triangle is one, since it is acute-angled only; but each of the rest is triple.

For the isosceles is either right-angled, or obtuse-angled, or acute-angled; and, in like manner, the scalene triangle possesses this triple difference. If then, these have a triple distinction, but the equilateral has but one mode of existence, all the species of triangle will be seven. But again, you will understand the proportion of triangles to the things which are, according to the division of sides; for the equilateral, entirely excelling in equality and simplicity, is allied to divine souls; since it is the measure and equality of things unequal, in the same manner as divinity of all inferior concerns. But the isosceles triangle is allied to the better genera, which govern a material nature, the greater part of which genera is held by the limitation of measure; but their extremes extend to inequality and material immoderation; for the two sides of an isosceles triangle are equal, but the base is unequal. But a scalene triangle symbolizes with partible lives, which are on all sides lame and defective, which prepare themselves for generation, and are replete with matter and material imperfection.

## DEFINITION XXX.

Of Quadrilateral Figures, a QUADRANGLE or SQUARE is that which has all its Sides equal, and all its Angles Right Angles.

## DEFINITION XXXI.

An OBLONG is that which has all Angles right Angles, but has not all its Sides equal.

## DEFINITION XXXII.

A rhombus, is that which has all its Sides equal, but its Angles are not right Angles.

## DEFINITION XXXIII.

A rhomboid is that which has its opposite Sides equal to one another, but all its Sides are not equal, nor its Angles Right Angles.

## DEFINITION XXXIV.

All other Quadrilateral Figures besides these, are called TRAPEZIUMS.

It is requisite that the first division of quadrilateral figures should take place in two numbers; and that some of them should be called parallelograms, but others non-parallelograms. But of parallelograms some are rectangular and equilateral, as quadrangles; but others neither of these, as rhomboids: others again, are rectangular, but not equilateral, as oblongs: but others, on the contrary, are equilateral, but not rectangular, as the rhombuses. For it is requisite either to possess both, viz. equality of sides and rectitude of angles, or neither; or one of these, and this in a twofold respect. Hence a parallelogram has a quadruple subsistence. But of non-parallelograms, some have only two parallel sides, and not the rest; but others have none of their sides parallel. And those are called Trapeziums, but these Trapezoids. But of Trapeziums, some, indeed, have the sides equal, by which the parallel sides of this kind are conjoined; but others unequal: and the former of these are called isosceles trapeziums; but the latter scalene trapeziums.

A quadrilateral figure, therefore, is constituted by us according to a seven-fold distribution. For one is a quadrangle; but the other is an oblong; the third a rhombus; the fourth a rhomboides; the fifth an isosceles trapezium; the sixth a scalene trapezium; the seventh a trapezoid. But Possidonius makes a perfect division of right-lined quadrilateral figures into so many members; for he establishes seven species of these; as likewise of triangles. But Euclid could not divide into parallelograms and non parallelograms, because he neither mentions parallels, nor teaches us concerning the parallelogram itself. But trapeziums, and all trapezoids, he calls by a common name, describing trapeziums themselves, according to the difference of those four figures, in which the property of parallelograms is verified. And this is to have the opposite sides and angles equal.

For a quadrangle and an oblong, and a rhombus, have their opposite sides and angles equal. But in a rhomboides he only adds this, that its opposite sides are equal, lest he should define it by negations alone, since he neither calls it equilateral, nor rectangular. For where we want proper appellations, it is necessary to use such as are common. But we should hear Euclid shewing that this is common to all parallelograms. But a rhombus appears to be a quadrangle having its sides moved, and a rhomboides a moved oblong. Hence, according to sides, these do not differ from those; but they vary only according to the obtuseness and acutenes of angles; since the quadrangle and the oblong are rectangular. For if you conceive a quadrangle or an oblong, having its sides drawn in such a manner, that while two of its opposite angles are dilated, the other two are contracted; then the dilated angles will appear obtuse, and the contracted, acute.

And the appellation of rhombus seems to have been imposed from motion. For if you conceive a quadrangle moving after the manner of a rhombus, it will appear to you changed in order, according to its angles: just as if a circle is moved after the manner of a sling, it will immediately exhibit the appearance of an ellipsis. But here you may perhaps enquire concerning the quadrangle, which it has this denomination? and why the application of quadrangle may not be applied to other quadrilateral figures, as the name of triangle is common to all those which are neither equiangular nor equilateral, and in like manner of quinquangles or pentagons; for the geometrician, in these, adds only the particle an equilateral triangle, or a quinquangle, which is equilateral and equiangular, is if these could not be otherwise than such as they are? But when he mentions a quadrangle, he immediately indicates that it must be equilateral and rectangular.

But the reason of this is as follows: a quadrangle alone has the best space, both according to its sides and angles. For each of the latter is right, intercepting a measure of angles, which neither receives intention nor remission. As it excels, therefore, in both respects, it deservedly obtains a common appellation. But a triangle, though it may have equal sides, yet will in this case have all its angles acute, and a quinquangle all its angles obtuse. Since, therefore, of all quadrilateral figures, a quadrangle alone is replete with equality of sides, and rectitude of angles, it was not undeservedly allotted this appellation: for, to excellent forms, we often dedicate the name of the whole. But it appeared also to the Pythagoreans, that this property of quadrilateral figures, principally conveyed an image of a divine essence. For they particularly signified by this, a pure and immaculate order.

Since rectitude imitates inflexibility, but equality a firm and permanent power: for motion emanates from inequality, but quiet from equality itself. The gods, therefore, who are the authors to all things of stable disposition, of pure and uncontaminated order, and of indeclinable power, are deservedly manifested as from an image, by a quadrangular figure. But, besides these, Philolaus also, according to another apprehension, calls a quadrangular angle, the angle of Rhea, Ceres and Vesta. For, since a quadrangle constitutes the earth, and is its proximate element, as we learn from Timæus, but the earth herself receives from all these divinities, genital feeds, and prolific powers, he does not unjustly consecrate the angle of a quadrangle to these goddesses, the bestowers of life. For some call both the earth and Ceres, Vesta, and they say that Rhea totally participates her nature, and that all generative causes are contained in her essence.

Philolaus, therefore, says that a quadrangular angle comprehends, by a certain terrestial power, one union of the divine genera. But some assimilate a quadrangle to universal virtue, so far as every quadrangle from its perfection has four right angles. Just as we say that each of the virtues is perfect, content with itself, the measure and bound of life, and the middle of every thing which, in morals, corresponds to the obtuse and acute. But it is by no means proper to conceal, that Philolaus attributes a triangular angle to four, but a quadrangular angle to three gods, exhibiting their alternate transition, and the community of all things in all, of odd natures in the even, and of even in the odd. Hence, the tetradic ternary, and the triadic quaternary, participating of prolific and efficacious goods, contain the whole ornament of generable natures, and preserve them in their proper state.

From which the duodenary, or the number twelve, is excited to a singular unity, viz. the government of Jupiter. For Philolaus says, that the angle of a dodecagon (or twelve-sided figure) belongs to Jove, so far as Jupiter contains and preserves, by his singular union, the whole number of the duodenary. For also, according to Plato, Jupiter presides over the duodenary, and governs and moderates the universe with absolute sway. And thus much we have thought proper to discourse concerning quadrilateral figures, as well declaring the sense of our author, as likewise affording an occasion of more profound inspections to such as desire the knowledge of intelligible and occult essences.

## DEFINITION XXXV.

Parallel right lines are such as being in the same Plane, and produced both ways infinitely, will in no part mutually coincide.

What the elements of parallels are, and by what accidents in these they may be known, we shall afterwards learn: but what parallel right lines are, he defines in these words: "It is requisite, therefore (says he), that they should be in one plane, and while they are produced both ways have no co-incidence, but be extended in infinitum." For non-parallel lines also, if they are produced to a certain distance, will not coincide. But to be produced infinitely, without coincidence, expresses the property of parallels. Nor yet this absolutely, but to be extended both ways infinitely, and not coincide. For it is possible that non-parallel lines may also be produced one way infinitely, but not the other; since, verging in this part, they are far distant from mutual coincidence in the other. But the reason of this is, because two right-lines cannot comprehend space; for if they verge to each other both ways, this cannot happen.

Besides this, he very properly considers the right-lines as subsisting in the same plane. For if the one should be in a subject plane, but the other in one elevated, they will not mutually coincide according to every position, yet they are not on this account parallel. The plane, therefore, should be one, and they should be produced both ways infinitely, and not coincide in either part. For with these conditions, the right-lines will be parallel. And agreeable to this, Euclid defines parallel right-lines. But Possidonius says, parallel lines are such as neither incline nor diverge in one plane; but have all the perpendiculars equal which are drawn from the points of the one to the other. But such lines as make their perpendiculars always greater and less, will some time or other coincide, because they mutually verge to each other. For a perpendicular is capable of bounding the altitudes of spaces, and the distances of lines.

On which account, when the perpendiculars are equal, the distances of the right lines are also equal; but when they are greater and less, the distance also becomes greater and less, and they mutually verge in those parts, in which the lesser perpendiculars are found. But it is requisite to know, that non-coincidence does not entirely form parallel lines. For the circumferences of concentric circles do not coincide: but it is likewise requisite that they should be infinity produced. But this property is not only inherent in right, but also in other lines: for it is possible to conceive spirals described in order about right lines, which if produced infinitely together with the right lines, will never coincide. Geminus, therefore, makes a very proper division in this place, affirming from the beginning, that of lines some are bounded, and contain figure, as the circle and ellipsis, likewise the cissoid, and many others; but others are indeterminate, which may be produced infinitely, as the right-line, and the section of a right-angled, and obtuse-angled cone; likewise the conchoid itself.

But again, of those which may be produced in infinitum, some comprehend no figure, as the right-line and the conic sections; but others, returning into themselves, and forming figure, may afterwards be infinitely produced. And of these some will not hereafter coincide, which resist coincidence, how far soever they may be produced; but others are coincident, which will some time or other coincide. But of non-coincident lines, some are mutually in one plane; and others not. And of non-coincidents subsisting in one plane, some are always mutually distant by an equal interval; but others always diminish the interval, as an hyperbola in its inclination to a right line, and likewise the conchoid. For In this curve it is manifest on account of the right line P O, cutting the rule in H that the point o will never arrive at rule CH; but because h O is perpetually equal to C A, and the angle of section is continually more acute, the distance of the point O from C H will at length be less than any given distance, and consequently the right line C H will be an assymptote [sic.] to the curve AO.

When the pole is at P, so that P C is equal to C A, the choncoid A O described by the revolution of P A, is called a primary conchoid, and those described from the poles p, and n, or the curves A o, A w, secondary conchoids; and these are either contracted or protracted, as the excentricity P C, is greater or less than the generative radius C A, which is called the altitude of the curve.

Now, from the nature of the conchoid, it may be easily inferred, that not only the exterior conchoid A w will never coincide with the right line C H, but this is likewise true of the conchoids AO, A o; and by infinitely extending the right-line An, an infinite number of conchoids may be described between the exterior conchoid A w, and the line CH, no one of which shall ever coincide with the assymptote C H. And this paradoxical property of the conchoid which has not been observed by any mathematician, is a legitimate consequence of the infinite divisibility of quantity. Not, indeed, that quantity admits of an actual division in infinitum, for this is absurd and impossible; but it is endued with an unwearied capacity of division, and a power of being diffused into multitude, which can never be exhausted. And this infinite capacity which it possesses arises from its participation of the indefinite duad; the source of boundless diffusion, and innumerable multitude.

But this singular property is not confined to the conchoid, but is found in the following curve. Conceive that the right line AC which is perpendicular to the indefinite line X Y, is equal to the quadrantal arch H D, described from the centre C, with the radius C D: then from the same centre C, with the several distances CE, CF, CG, describe the arches El, Fn, Gp, each of which must be conceived equal to the first arch HD, and so on infinitely.

Now, if the points H, k, l, n, p, be joined, they will form a curve line, approaching continually nearer to the right line A B (parallel to C Y) but never effecting a perfect coincidence. This will be evident from considering that each of the sines of the arches HD, l E, n F, &c. being less than its respective arch, must also be less than the right-line A C, and consequently can never coincide with the right-line A B.

But if other arches D i, E m, F o &c. each of them equal to the right line A C, and described from one centre, tangents to the former arches HD, lE, nF, &c. be supposed; it is evident that the points H, i, m, o, &c. being joined, will form a curve line, which shall pass beyond the former curve, and converge still nearer to the line A B, without a possibility of ever becoming coincident: for since the arches D i, E m, F o, &c. have less these though they always diminish the interval, never coincide. And they mutually converge, indeed, but never perfectly nod to each other; which is indeed a theorem in geometry especially admirable, exhibiting certain lines endued with a non-assenting nod. But the right-lines, which are always distant by an equal interval, and which never diminish the space placed between them in one plane, are parallel lines. And thus much we have extracted from the studies of the elegant Geminus, for the purpose of explaining the present definition.